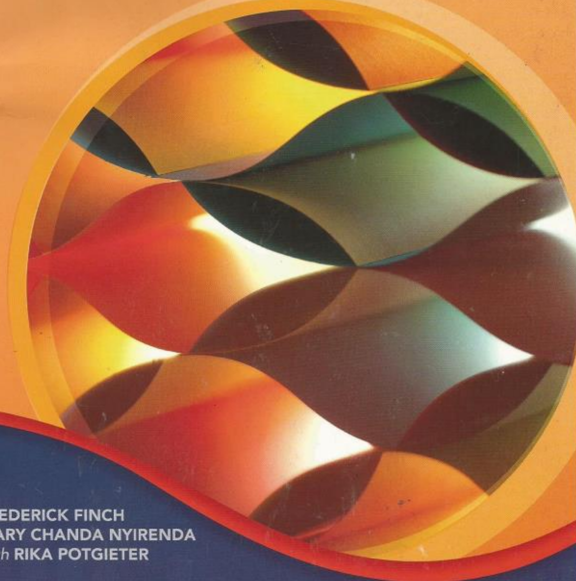
OXFORD

Progress in Mathematics 10

LEARNER'S BOOK



FREDERICK FINCH
MARY CHANDA NYIRENDA
with RIKA POTGIETER

05

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Progress in Mathematics

Grade 10

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OXFORD
UNIVERSITY PRESS

Contents

How to use this b

Welcome to the *Progress*
This series is based o
Ministry of Education,
knowledge, skills and v
in Mathematics Grade 1
success in this subject.
This page will help y
The book is divided i
covered in your Mathe
On the first page of e

The **topic summary** will help you to revise key learning points in the topic quickly.

Revision exercises help you revise the topic's work and check your understanding.

Assessment exercises help you prepare for tests and exams.

How to use this book

Welcome to the *Progress in Mathematics* series for Grades 10–12!

This series is based on the *Senior Secondary Syllabus for Mathematics* issued by the Ministry of Education, Science, Vocational Training and Early Education. All the knowledge, skills and values expressed in the document are addressed in *Progress in Mathematics Grade 10 Learner's Book*, so that you can feel confident about your success in this subject.

This page will help you understand how the book works.

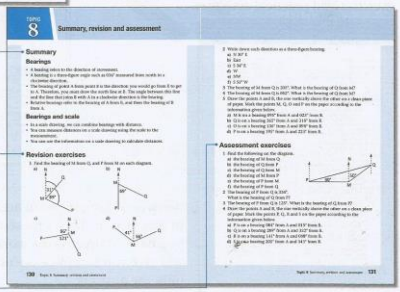
The book is divided into topics so that you can easily see what content will be covered in your Mathematics class.

On the first page of every topic, you will find:

A **table** of sub-topics and specific outcomes that will be covered in the topic.

A **starter activity** helps to introduce the topic with knowledge you already have.

At the end of each topic, you will find the following:

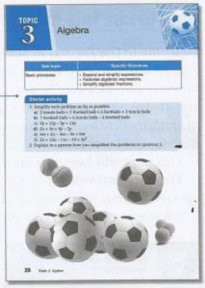


The **topic summary** will help you to revise key learning points in the topic quickly.

Revision exercises help you revise the topic's work and check your understanding.

Assessment exercises help you prepare for tests and exams.

You will see the following throughout the book:



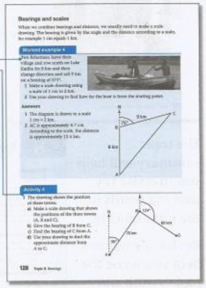
The **starter activity** prepares you for the topic you are about to start.

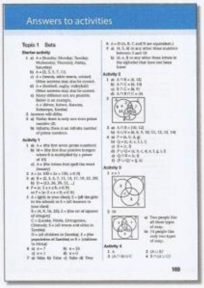
New words boxes give you the definitions of key words or explain what a certain new word means. These words and the definitions are also in the glossary at the back of the book.

Did you know? boxes give you more and new knowledge about what you are learning.

Worked examples give an example with a model answer that shows step by step how to do a calculation.

Activities are tasks where you apply the knowledge and skills you learnt in a section. *Note:* We use the term *activity* to refer to written exercises and practical activities.





Answers for questions in activities are given in this section. This is for self-assessment once you have completed the work. Your teacher will give you complete calculations and not only the answers.

TOPIC 1 Se

Sub-topic

Set operations

Starter activity

1 List the element
a) the seven days
b) the first five p
c) three different

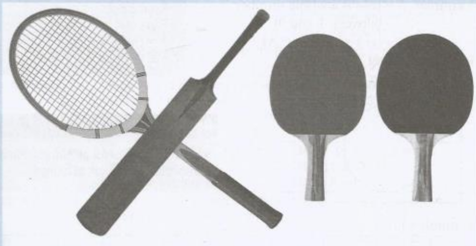
d) three different
e) five Zambian
2 Speak to two clas
celebrate your bi
3 We can count all
infinite set. Look
even prime numb
of the set of prim
Explain each ans
a) even prime nu
b) uneven prime

TOPIC 1 Sets

Sub-topic	Specific Outcomes
Set operations	• Carry out operations on sets. • Apply higher operations on sets.

Starter activity

1 List the element(s) for each set. Give your answers in the notation: A = {...}.
   a) the seven days of the week
   b) the first five prime numbers
   c) three different types of sport that are played with a racket or a bat



   d) three different types of sport that are not played with a racket or a bat
   e) five Zambian towns that start with a K
2 Speak to two classmates and list the months in which the three of you celebrate your birthdays.
3 We can count all the elements in a finite set, but not all the elements in an infinite set. Look at your set of prime numbers in question 1b). The set of even prime numbers and the set of uneven prime numbers are both subsets of the set of prime numbers. Are the following subsets finite or infinite? Explain each answer.
   a) even prime numbers
   b) uneven prime numbers

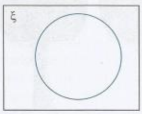
SUB-TOPIC 1 Set operations

Introduction

We often group objects to help us manage different aspects of our lives. For example, we stack plates in a cupboard, pack books on a shelf, hang shirts on coathangers in a cupboard and keep money in a wallet. For example, you would not pack your shoes on a shelf with the breakfast bowls or put your money in the grocery cupboard. These groups are like sets.

There are four ways to represent a set:

- List the elements in a set
  Example: N = {1, 2, 3, 4, 5, 6, 7, 8, 9}
- Describe the elements in a set
  Example: N = {the set of natural numbers (ℕ) smaller than 10}
- Use set builder notation
  Examples: A = {*x*: *x* is a whole number between 1 and 9}
  N = {x: $1 \leq x \leq 9$, $x \in \mathbb{N}$}
- Represent the set in diagrams:
  » Venn diagram



  » number line

Remember

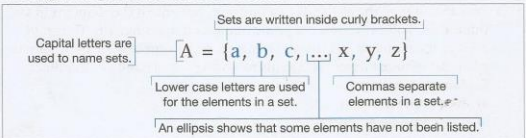
- A finite set has a countable number of elements.
  Example: the days of the week
- An infinite set has no limits or boundaries – you cannot count the number of elements in an infinite set.
  Example: integers
  ℕ = {1, 2, 3, …}
  ℤ = {…, −3, −2, −1, 0, 1, 2, 3, …}

Remember

- ℕ is the symbol for all natural numbers.
- ℤ is the symbol for all integers.

Set notation

The diagram explains the notation we use to write sets.



Carry out ope

Operations on sets inc
two or more sets.
Operations on sets,
summarised in the ta

Term and symbol
element (∈)
not element (∉)
subset (⊂)
not a subset (⊄)
empty set (∅ or { })
number of elements in a set (n)
equal sets (=)
equivalent sets (≡)
not equal sets (≠)
universal (U) set (ξ) (entirety)
intersection (∩)
union (∪)
complement (A')

Carry out operations on sets

Operations on sets include finding the union, intersection and complement of two or more sets.

Operations on sets, and terms and symbols we use when working with sets are summarised in the table.

Term and symbol	Description	Examples
element (∈)	An element belongs to a set.	dog ∈ {mammals} a ∈ {a, b, c}
not element (∉)	An element does not belong to a set	chicken ∉ {mammals} a ∉ {b, c, d}
subset (⊂)	Set A is a subset of set B if each element of A is also an element of B. Each set is a subset of itself.	{men} ⊂ {humans} {a, b} ⊂ {a, b, c, d} A ⊂ A
not a subset (⊄)	Set A is not a subset of set B if at least one element in A is not an element of B.	birds ⊄ {mammals} {p, q, r} ⊄ {a, b, c, d} {a, b, e} ⊄ {a, b, c, d}
empty set (∅ or { })	A set is an empty set if it contains no elements.	{chickens with teeth} = { } {coastal towns in Zambia} = ∅
number of elements in a set (n)	The total number of elements in a set	If A = {p, q, r, s}, then n(A) = 4
equal sets (=)	Sets A and B are equal if all elements of A are also elements of B. (The order of elements does not matter.)	A = B and B = A {5, 3, 1} = {3, 1, 5}
equivalent sets (≡)	Two sets are equivalent if they contain the same number of elements.	If A = {a, b, c, d} and B = {1, 2, 3, 4}, then A ≡ B
not equal sets (≠)	Two sets are not equal if one set contains an element the other set does not contain.	{a, b, c} ≠ {a, b, c, d}
universal (U) set (ξ) (entirety)	This set contains all the elements in a certain context.	ξ = {a, e, i, o, u} ξ is the universal set of vowels in English. So, a ∈ ξ and b ∉ ξ.
intersection (∩)	The intersection between two sets gives the elements that are members of both sets.	{a, b, c} ∩ {b, d} = {b} {1, 2, 3} ∩ {–1, 0, 1, 2} = {1, 2}
union (∪)	A union of two sets contains all the elements (members) of both sets.	{a, b, c} ∪ {b, d} = {a, b, c, d} {1, 2, 3} ∩ {–1, 0, 1, 2} = {–1, 0, 1, 2, 3}
complement (A')	The complement of a set contains all the elements of its universal set that are not elements of set A.	If all the numbers in a universal set = {1, 2, 3, 4, 5, 6} and A = {1, 2, 3}, then A' = {4, 5, 6}

Revise your knowledge of sets as you work through Activity 1. Refer to the table on page 3 if necessary.

Activity 1

1 Describe each set in words.
  a) A = {2, 3, 5, 7, 11, 13, 17}
  b) M = {80, 800, 8 000, 80 000}
  c) A = {j, a, n, u, a, r, y}
2 Represent even numbers between 100 and 150 in set builder notation.
3 List the elements of each set.
  a) B = {$0 \leq x \leq 30$, $x \in$ prime numbers}
  b) List the set D = {x: $x = 13p$, where $p = 1, 2, 3, 4, \ldots$}
4 Use set builder notation to describe the following set.
  P = {3, 4, 5, 6, 7, 8}
5 Suggest a universal set for each set.
  A = {girls in your class}
  B = {4, 9, 16, 25}
  C = {Lusaka, Ndola, Livingstone, Chinsali}
  D = {all children in Zambia}
6 Find the number (n) of elements in each set.
  a) {1, 2, 3, 4, 5, 6, 7} b) {letters of the alphabet}
  c) {x: $90 < x < 100$; x is a prime number}
  d) {months of the year beginning with the letter P}
7 Which statements are true and which are false if B = {x: $x > 1$}?
  a) $1 \in B$ b) $n(B) = 1$ c) $1\,000 \notin B$ d) $0 \notin B$
8 Which sets are equal and which sets are equivalent?
  A = {a, b, c, d, e} B = {1, 2, 3, 4, 5} C = $\{\frac{1}{2}, \frac{2}{1}, \frac{5}{1}, \frac{3}{1}, \frac{4}{1}\}$
  D = {b, e, a, c, d} E = {letters of the alphabet}
9 List any three elements that have not been listed in each set.
  a) A = {1, 2, 3, … 10} b) A = {a, b, c, …}

Numerical problems involving sets

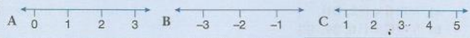
In this section, you will revise the operations on sets: union, intersection and complement. You will then look at using Venn diagrams and identifying the number of elements in a set.

Union, intersection and complement of sets

A union of two or more sets contains all the elements (members) of the sets. The intersection between two sets gives the elements that are members of both sets. All the elements that are not elements of set A are the complement (A') of set A. To find A', you must remove all elements in A from all elements in the universal set.

Worked example 1

1 Give the union (∪) of the sets.
   a) A = {10, 20, 30, 40} and B = {5, 15, 25, 35, 45}
   b) A = {a, b, c}, B = {x, y, z} and C = {1, 2}
   c) A = {x: $x \in$ N, x is a prime number smaller than 20} and B = {1, 2, 3, 4, 5, 6}
   d) A = {multiples of 3 larger than 1 and smaller than 20}, B = {multiples of 4 larger than 1 and smaller than 20} and C = {multiples of 10 larger than 1 and smaller than 20}
   e)



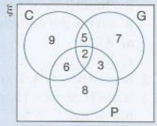
2 Three friends who share accommodation go to a market. Kasuba wants to buy eggs, Mumbi wants to buy milk and eggs, Alinani wants to buy bread and milk. What should the three friends buy to ensure that everyone is happy with their purchases? Explain.

3 Give the intersection (∩) of the sets.
   a) A = {0, 1, 2, 3, 5, 6} and B = {–3, –2, –1, 0, 1, 2}
   b) A = {0, 1, 2, 3, 5}, B = {– 3, –2, –1, 0, 1} and C = {0, 1, 2, 3, 4, 5}

4 A = {multiples of 3 larger than 1 and smaller than 40}, B = {multiples of 4 larger than 1 and smaller than 40} C = {multiples of 8 larger than 1 and smaller than 40}
   a) Give A ∩ B.
   b) Find B ∩ C.
   c) Give A ∩ B ∩ C.

5 Give the complement of set A if E = {0, 1, 2, 3, 5, 6} and A = {0, 1, 2}.

6 Look at the Venn diagram.
   a) Give the complement of each set (C, G and P).
   b) List (G ∩ P)'.



Answers

1 a) A ∪ B = {5, 10, 15, 20, 25, 30, 35, 40, 45}
   b) A ∪ B ∪ C = {a, b, c, x, y, z, 1, 2}
   c) A = {2, 3, 5, 7, 11, 13, 17, 19} and B = {1, 2, 3, 4, 5, 6}
      A ∪ B = {1, 2, 3, 4, 5, 6, 7, 11, 13, 17, 19}
   d) A = {3, 6, 9, 12, 15, 18}, B = {4, 8, 12, 16} and C = {10}
      A ∪ B ∪ C = {3, 4, 6, 8, 9, 10, 12, 15, 16, 18}
   e) A = {0, 1, 2, 3}, B = {–3, –2, –1} and C = {1, 2, 3, 4, 5}
      A ∪ B ∪ C = {–3, –2, –1, 0, 1, 2, 3, 4, 5}

Worked example 1 (continued)

2 Kasuba = {eggs}, Mumbi = {milk, eggs} and Alinani = {bread, milk}
They should buy eggs, milk and bread so that they will all have the items they wanted.

3 a) A ∩ B = {0, 1, 2}
b) A ∩ B ∩ C = {0, 1}

4 A = {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39},
B = {4, 8, 12, 16, 20, 24, 28, 32, 36} and C = {8, 16, 24, 32}
a) A ∩ B = {12, 24}
b) B ∩ C = {16, 24, 32}
c) A ∩ B ∩ C = {24}

5 E – A = {3, 5, 6}

6 E = {2, 3, 5, 6, 7, 8, 9}
C = {2, 5, 6, 9}, G = {2, 3, 5, 7} and P = {2, 3, 6, 8}
a) C' = E – C = {3, 7, 8}
G' = E – G = {6, 8, 9}
P' = E – P = {5, 7, 9}
b) (G ∩ P)' = E – (G ∩ P)
= {2, 3, 5, 6, 7, 8, 9} – {2, 3}
= {5, 6, 7, 8, 9}

Remember to refer to the table on page 3 when answering the questions in the activity that follows.

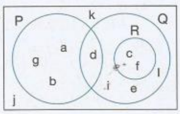
Activity 2

1 If E = {whole numbers from 1 to 15}, A = {even numbers between 1 and 15}, B = {multiples of 3 between 1 and 15} and C = {6, 7, 9, 10}, list the sets A and B and list the elements in each solution.
a) A ∩ B
b) A ∩ C
c) B ∩ C
d) A ∩ B ∩ C

2 Represent sets A, B and C from question 1 on a Venn diagram.

3 If A = {x: 9 < x < 15, x is a whole number} and B = {6, 8, 10, 12}, find the following.
a) A ∩ B
b) A ∪ B

4 Refer to the Venn diagram. List the elements in each solution.
a) P
b) Q
c) R
d) P ∪ Q
e) Q ∩ R
f) (P ∪ Q)'



Venn diagrams

A Venn diagram shows a
of sets. We can use Venn
If the universal set has n
diagram so that you can
set.
For example, the Venn d
universal set and three s
E = {1, 2, 3, 4, 5, 6, 7, 8,
A = {2, 4, 6, 8}
B = {1, 2, 3, 5, 6, 7}
C = {3, 5}

Number of elem

We write the number of
n(A) = 5. There are five e
add the number of elem
union of the sets. The fo

Worked example 2

1 In a class of 25 lea
both Chemistry an
2 A group of girls we
like cola (C), 7 like
Find the number o
3 In a group of 10 le
every day. How ma
bicycle on the oth

Answers

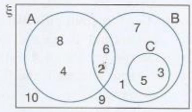
1 Number of learner
Number of learner
Number of learner
The number of lea
both Chemistry an
(14 – 4 = 10) and
The Venn diagram
who study Chemi
Note: Unlike the a
right shows the nu
and Physics and no

Venn diagrams

A Venn diagram shows all the possible logical relations between finite collections of sets. We can use Venn diagrams to solve problems and explain solutions clearly. If the universal set has not been defined clearly, it often helps to draw a Venn diagram so that you can see what which elements are included in the universal set.

For example, the Venn diagram shows the universal set and three sets:

E = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {2, 4, 6, 8}
B = {1, 2, 3, 5, 6, 7}
C = {3, 5}



Number of elements in sets

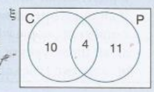
We write the number of elements in a set as $n(A)$. If A = {a, b, c, d, e}, then $n(A) = 5$. There are five elements in set A. When sets intersect, you cannot just add the number of elements in both sets to find the number of elements in a union of the sets. The following example illustrates this.

Worked example 2

1 In a class of 25 learners, 14 study Chemistry, 15 study Physics and 4 study both Chemistry and Physics. Illustrate this information on a Venn diagram.
2 A group of girls were asked about their favourite drink. Six girls said they like cola (C), 7 like apple juice (A) and 3 liked both cola and apple juice. Find the number of girls who took part in the survey.
3 In a group of 10 learners, 4 walk to school and 3 go to school by bicycle every day. How many learners walk to school on some days and go by bicycle on the other days? Use a Venn diagram to represent the situation.

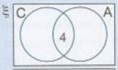
Answers

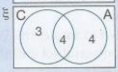
1 Number of learners who study Chemistry: 14
Number of learners who study Physics: 15
Number of learners who study both Chemistry and Physics: 4
The number of learners who study Chemistry includes those who study both Chemistry and Physics. Therefore, 10 learners study Chemistry only ($14 - 4 = 10$) and 11 learners study Physics only ($15 - 4 = 11$).
The Venn diagram represent the number of learners who study Chemistry and Physics.
Note: Unlike the above Venn diagrams, the one on the right shows the *number* of learners who take Chemistry and Physics and not the actual elements in a set.



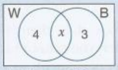
Worked example 2 (continued)

2 Draw two circles – one for cola and one for apple juice. The *number* of girls who like both drinks is shown in the first diagram. The six girls who like cola include the three who like both types of drink. Therefore, three girls like cola only (6 – 3 = 3). The seven girls who like apple juice include those who like both drinks. Therefore, four girls like only apple juice (7 – 3 = 4). The second diagram shows how many girls took part in the survey: 3 + 3 + 4 = 10.





3 W represents walk and B going to school by bicycle. Let x represent the *number* of learners who sometimes walk and at other times go to school by bicycle.



$4 + 3 + x = 10$

$x = 10 - 7$

$= 3$

Three learners sometimes walk to school and at other times go by bicycle.

Activity 3

1 In a class of 30 learners, 17 play football, 15 learners play volleyball, 12 learners play rugby, 9 learners play volleyball only and 3 learners play rugby only. All learners play at least one of the three sports.
Use a Venn diagram to find the number of learners who play only volleyball and rugby.

2 At a shop 40 people were asked which soap(s) they prefer – Clean, Great or Perfect.
Their responses show:
9 prefer Clean only
7 prefer Great only and 8 prefer Perfect only
7 prefer both Clean and Great
5 prefer both Great and Perfect
8 prefer Clean and Perfect.
If all those interviewed prefer at least one of the three types of soap, find the number of people who prefer:
a) all three types of soap
b) only two types of soap.

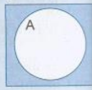
Shading Venn d

We can use shading to

Worked example 3

Use symbols to descr

1



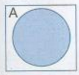
Answers

1 A'

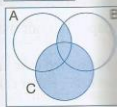
Activity 4

Describe the shaded a
in Worked example 3.

1



3



Practical activity

Choose three of the c
choice later).

1 Pick the four cha
important for a p
Illustrate your gr

2 Which three cha
chose?

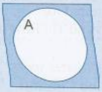
Characteristics: sens
friendly, good listeni
practical, good drive
Careers: taxi driver,
accountant, police of

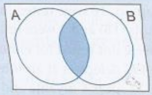
Shading Venn diagrams

We can use shading to show where elements are situated on a Venn diagram.

Worked example 3

Use symbols to describe the shaded area in each diagram.

1


2


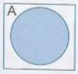
Answers

1 A'

2 A ∩ B

Activity 4

Describe the shaded area in each diagram in the same way such areas are described in Worked example 3.

1


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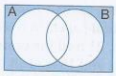

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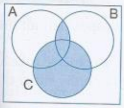

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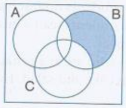


Practical activity

Choose three of the careers listed below that interest you (you may change your choice later).

1 Pick the four characteristics from the list below that you think are most important for a person who wants to be successful in each career you chose. Illustrate your group's decision on a Venn diagram.

2 Which three characteristics listed are not important for each career you chose?

Characteristics: sense of humour, good mathematical skills, hardworking, honest, friendly, good listening skills, good presentation skills, intelligent, problem-solver, practical, good driver, good at working with people

Careers: taxi driver, dress maker, teacher, personal assistant (PA), cupboard maker, accountant, police officer, lawyer

Apply higher operations on sets

In the next activity, you have use everything you have learnt in this topic about operations on sets (including union, intersection and complement) to answer the questions.

Activity 5

1 Of 35 learners, 16 were tested for HIV and AIDS, 10 were tested for tuberculosis (TB) and 7 were tested for both HIV and AIDS and TB. How many learners were not tested for either infection?

2 There are 197 delegates at an International Trade Fair; 85 delegates speak Icibemba, 74 speak Cinyanja and 15 speak both Icibemba and Cinyanja.
   a) How many delegates speak Icibemba but not Cinyanja?
   b) How many delegates speak Cinyanja but not Icibemba?

3 A survey of 150 people revealed that 121 people watch the early evening TV news broadcast, 64 watched the noon news broadcast and 47 watched both news broadcasts. How many did not watch either news broadcast?

4 A survey of 120 college students produced these results: 40 students read a business journal, 48 read a local paper, 70 read the campus paper, 25 read a business journal and a local paper, 28 read a local paper and the campus journal, 21 read the campus journal and a business journal and 18 read all three papers.

   a) How many students do not read any of the papers?
   b) How many students read a business journal and local paper, but not the campus paper?

5 Of 20 students who ate at a restaurant, 14 ordered salad, 10 ordered cake and 4 ordered both cake and salad. How many students did not order either cake or salad?

6 At a school, 100 learners were asked which sport they play. The results showed that 50 play football, 48 play basketball, 54 play tennis, 24 play football and basketball, 22 play basketball and tennis, 25 play football and tennis and 14 play all three sports.
   a) How many learners play tennis only?
   b) How many learners play football and tennis, but not basketball?
   c) How many learners do not play football, basketball or tennis?

7 At a school, 600 learners were asked to choose their favourite colour(s) – red or blue. The results showed that 420 chose red, 352 chose blue and 20 did not choose red or blue.
   a) How many learners chose both colours?
   b) How many learners chose red, but not blue?

8 Of 300 people who were tested for HIV and AIDS, 282 tested negative and the rest tested positive. Use a formula to find how many people tested positive for HIV and AIDS.

Summary

Operations on s

- A union of two or m
- The intersection betw both sets.
- All the elements that To find A', remove al

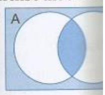
Revision exer

1 The Venn diagram Find each number elements in the se
   a) $n(A)$
   c) $n(C)$
   e) $n(A \cap C)$
   g) $n(B \cap C)'$

2 If A = {a, b, c, d, e} are false? Explain.
   a) A = B
   c) A ≡ B
   e) $A \cap B = \{a, e\}$

3 List the members o Venn diagram.
   a) A
   c) $A \cap B$
   e) A'
   g) $(A \cup B)'$

4 Describe the shade
   a)



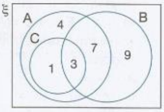
Summary

Operations on sets

- A union of two or more sets contains all the elements (members) of the sets.
- The intersection between two sets gives the elements that are members of both sets.
- All the elements that are not elements of set A are the complement (A') of set A. To find A', remove all elements in A from all elements in the universal set.

Revision exercises

1 The Venn diagram shows sets A, B and C. Find each number (n) of elements (not all the elements in the sets).

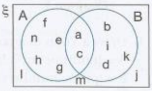


a) $n(A)$ b) $n(B)$
c) $n(C)$ d) $n(A \cap B)$
e) $n(A \cap C)$ f) $n(C)'$
g) $n(B \cap C)'$ h) $n(A \cap B \cap C)$

2 If A = {a, b, c, d, e} and B = {a, e, i, o, u}, which statements are true and which are false? Explain.

a) A = B b) $A \subset B$
c) A ≡ B d) $a \in A$ and $a \in B$
e) $A \cap B = \{a, e\}$ f) $A \cup B = \{a, b, c, d, e, i, o, u\}$

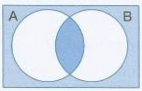
3 List the members of the following sets on the Venn diagram.



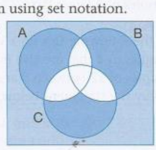
a) A b) B
c) $A \cap B$ d) $A \cup B$
e) A' f) $(A \cap B)'$
g) $(A \cup B)'$ h) $A' \cap B$

4 Describe the shaded area in each diagram using set notation.

a)

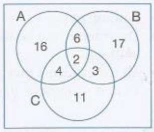


b)



5 In the Venn diagram, A = {learners who take Art}, B = {learners who take Biology} and C = {learners who take Chemistry}. Find the number of learners who take the following subjects or combination of subjects.

a) Art
b) Biology
c) both Biology and Chemistry
d) only Art
e) both Art and Biology, but not Chemistry
f) only two of the three subjects



6 Answer the questions for sets A and B.

a) If $n(A) = 9$, $n(B) = 5$ and $n(A \cap B) = 3$, find $n(A \cup B)$.
b) If $n(A \cup B) = 20$, $n(A) = 12$ and $n(B) = 10$, find $n(A \cap B)$.
c) $n(A \cup B) = 40$, $n(A \cap B) = 8$, $n(B) = 24$, find $n(A)$.

7 Make four copies of each diagram and use shading to show the following regions.

a) $A \cap B$
b) $(A \cup B)$
c) B'
d) $(A \cap B)'$

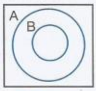


Diagram 1

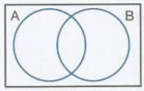


Diagram 2

8 Draw a Venn diagram to show the following sets:
A = {multiples of 3 larger than 1 and smaller than 30}
B = {prime numbers larger than 4 and smaller than 30}
C = {multiples of 5 larger than 29 and smaller than 50}
Find $A \cap B \cap C$.

9 Illustrate the following sets on a Venn diagram:
A = {multiples of 4 that are also multiples of 6 between 0 and 50}
B = {multiples of 12 between 0 and 50}
Explain why the statement below is true.
A = B

Assessment

1 Find the number
and unions.
a) $n(A \cap B \cap C)$
c) $n[(A \cup B) \cap C$
e) $n[(A \cup C)' \cap B$
g) $n[(A \cup B \cup C)'$

2 Which statement
a) If A = B, then
b) If M = {month

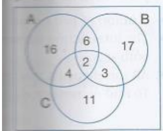
3 Out of 100 learn
• 49 mixed fruit
• 16 mixed fruit
• 18 both mixed
23 both mixed
All learners dran
a) Use x for the
copy of the V
necessary, exp
b) Give the follo
• the number
cola only
• the number
juice and c
• the number
c) Form an equa
drank all thre

4 The Venn diagra
If $n(A) = 24$, find
following.
a) $n(B)$
c) $n(A \cap B)$
e) $n[(A \cap B) \cup C$

5 Answer the ques
a) $n(A) = 30$, $n(B$
b) $n(A \cup B) = 28$
c) $n(A \cup B) = 20$

ontinued

learners who take
the number of learners
bjects.



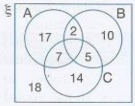
n B).

show the following

en 0 and 50}

Assessment exercises

1 Find the number of elements (n) in the intersections and unions.



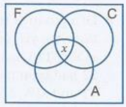
a) $n(A \cap B \cap C)$ b) $n(A \cup B)$
c) $n[(A \cup B) \cap C]$ d) $n[(A \cap B) \cup C]$
e) $n[(A \cup C)' \cap B]$ f) $n[(A \cup B)' \cap C]$
g) $n[(A \cup B \cup C)']$

2 Which statements are true and which are false?
a) If A = B, then set A is a subset of B.
b) If M = {months of the year} and L = {January, June, July}, then $M \subset L$.

3 Out of 100 learners at a party the following number chose each type of drink:
- 49 mixed fruit juice (F), 56 cola (C), 49 apple juice (A)
- 16 mixed fruit juice only, 20 cola only, 18 apple juice only
- 18 both mixed fruit juice and apple juice, 21 both cola and apple juice, 23 both mixed fruit juice and cola

All learners drank at least one drink.

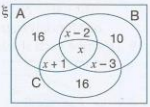
a) Use x for the number of learners who drank all three types of drink. Make a copy of the Venn diagram and complete it. Where necessary, express your answers in terms of x.



b) Give the following in terms of x:
- the number of learners who drank apple juice and cola only
- the number of learners who drank mixed fruit juice and cola only
- the number of learners who drank mixed fruit juice and apple juice only

c) Form an equation in x and solve it to find the number of learners who drank all three types of drink.

4 The Venn diagram shows sets A, B and C.
If $n(A) = 24$, find the value of x. Then find the following.



a) $n(B)$ b) $n(C)$
c) $n(A \cap B)$ d) $n(A \cup B)$
e) $n[(A \cap B) \cup C]$

5 Answer the questions for sets A and B.
a) $n(A) = 30$, $n(B) = 25$, $n(A \cap B) = 3$, find $n(A \cup B)$.
b) $n(A \cup B) = 28$, $n(A) = 15$, $n(A \cap B) = 10$, find $n(B)$.
c) $n(A \cup B) = 20$, $n(A) = 12$, $n(A \cap B) = 8$, find $n(B)'$.

Revision and assessment (continued)

6 A group of Zambian students were asked about which neighbouring countries, Malawi, Zimbabwe and Botswana they had visited. The following information was obtained:

- 80 had visited Malawi, 70 had visited Botswana, 55 had visited Zimbabwe
- 35 had visited both Malawi and Botswana, 30 had visited both Malawi and Zimbabwe, 30 had visited both Zimbabwe and Botswana
- 10 had visited all three countries
- 20 had not visited any of the three countries.

Find the number of students who took part in the survey.

7 Make four copies of each diagram and use shading to show the regions.

a) $A \cap B \cup C$ **b)** $(A \cup B)'$ **c)** $(A \cup B)' \cap C$ **d)** $(A \cap B)' \cup C$

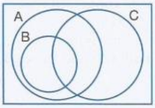


Diagram 1

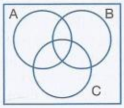
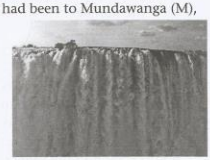


Diagram 2

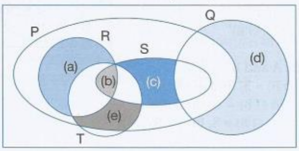
8 Of a group of tourists returning from Zambia, 200 were asked which places they had visited. Of these tourists, 148 had been to Kafue National Park (KNP), 116 had been to Victoria Falls (VF), 96 had been to Mundawanga (M), 82 had been to KNP and VF, 71 had been to VF and MW, 56 had been to KNP and MW, and 44 had been to all the three places.



a) How many tourists visited only Victoria Falls?

b) How many of the 200 tourists did not visit Kafue National Park, Victoria Falls or Mundawanga?

9 Describe each shaded area (a to e).



TOPIC 2 Inc

Sub-topic

Indices

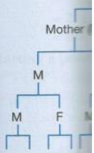
Starter activity

In Grade 8, you worke
knowledge.

1 Write each numbe
a) 8

2 Write each produ
memory about po
a) 2 × 2 × 2 × 2
d) 11 × 11 × 11

3 Work with a partn
theoretically the n
people from whon
have in the follow
a) the fifth genera
b) the twelfth ger



Inonge has two pa
great-grandparent

TOPIC

2 Index notation

Sub-topic	Specific Outcomes
Indices	• Apply laws of indices • Simplify positive, negative and zero indices • Simplify fractional indices • Solve equations involving indices

Starter activity

In Grade 8, you worked with index notation. Use this activity to check your knowledge.

1 Write each number as a product of its prime factors.

a) 8 b) 25 c) 49

2 Write each product as a power. See the diagram on page 16 to refresh your memory about powers.

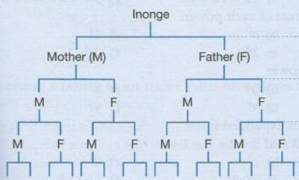
a) $2 \times 2 \times 2 \times 2$ b) $3 \times 3 \times 3$ c) $6 \times 6 \times 6 \times 6 \times 6$

d) $11 \times 11 \times 11$ e) $5 \times 5 \times 5 \times 5 \times 5 \times 5$ f) $7 \times 7 \times 7 \times 7$

3 Work with a partner. Look at the diagram below that shows how to estimate theoretically the number of ancestors a person has. (Your ancestors are the people from whom you are descended.) How many ancestors could Inonge have in the following number of generations?

a) the fifth generation before Inonge

b) the twelfth generation before Inonge

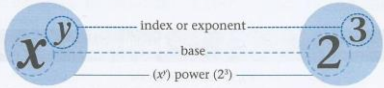


Inonge has two parents (2), four grandparents (2^2), eight great-grandparents (2^3) and ... great-great-grandparents.

Generation before you	Number
	1
First (parents)	$2 = 2$
Second (grandparents)	$4 = 2^2$
Third (great-grandparents)	$8 = 2^3$
Fourth (great-great-grandparents)	

SUB-TOPIC 1 Indices

The diagram shows two examples of a power.



The yth power of x, where x is the **base** and y is the **index**. You can only calculate this power if you have the values for x and y.

The third power of 2. The base is 2 and the index is 3. Say: base 2 raised to the power of 3, or 2 raised to the third power.

Definition

A power is a short cut for writing the repeated multiplication of a number or a product:
$a^m = a \times a \times a \times \ldots$ for m **factors**, where $m > 0$.
Example: $a^5 = a \times a \times a \times a \times a$
Compare the above with repeated addition
$ma = a + a + a + \ldots$ for m **terms**.
Examples:
- $2^4 = 2 \times 2 \times 2 \times 2$
- $4^2 = 4 \times 4$ Raising to the second power is called squaring (4^2 is 4 squared).
- $4^3 = 4 \times 4 \times 4$ Raising to the third power is called as cubing (4^3 is 4 cubed).

> **New words**
>
> **power (of a number):** the number raised to an index or exponent
> **index (or exponent):** the number of times a real number is multiplied by itself
> **base:** the number that is raised to an index
> **indices:** plural of index

Activity 1

1 Write the following as powers.
   a) 5×5 b) $5 \times 5 \times 5 \times 5$ c) $a \times a \times a \times a \times a$
   d) $2 \times 2 \times 2$ e) $12 \times 12 \times 12$ f) $3 \times 3 \times b \times b \times b \times b$
2 Give the index (exponent) of each power.
   a) c^2 b) a^0 c) 1^4
   d) 9^3 e) 2^2 f) 4^{-1}
3 Give the base of each power.
   a) a^3 b) b^1 c) 3^2
   d) 15^4 e) x^y f) 9^3
4 Write each expression in expanded form.
   Examples: $2^3 = 2 \times 2 \times 2$ and $3 \times 2 = 2 + 2 + 2$
   a) a^4 b) c^2 c) 2^5
   d) 10^4 e) $3(ab)$ f) $3x + y^6$
   g) $5^3 \times 2^2$ h) 4^3x^3 i) $(2x)^3$

Apply the laws

In Grade 8, you learnt abo
problems, we do certain o
BODMAS to remind you o
multiplication, addition a
applies to working with in
There are additional rul
(worked out) from the def
problems.

Laws of indices

The laws of indices are de
cuts for working with ind

Law 1: Multiplying powe
$a^m \times a^n = a \times a \times \ldots$ (for m
$= a \times a \times \ldots$ (for m
$= a^{m+n}$
Therefore, if the bases are
Example: $2^5 \times 2^3 = (2 \times 2$
$= 2 \times 2 \times 2 \times 2 \times$
$= 2^8$
$= 2^{5+3}$

Law 2: Dividing a power
$a^m \div a^n = \frac{a^m}{a^n} = \frac{a \times a \times \ldots}{a \times a \times \ldots}$
$= a \times a \times \ldots$ (for m
$= a^{m-n}$
Therefore, if the bases are
Example: $2^5 \div 2^3 = \frac{2^5}{2^3}$
$= \frac{2 \times 2 \times}{2 \times}$
$= 2^2$
$= 2^{5-3}$

Law 3: Raising a power
$(a^m)^n = (a^m) \times (a^m) \ldots$ (for
Example: $(a^2)^3 = (a^2) \times (a^2)$
$= (a \times a) \times$
$= a^6$
$= a^{2 \times 3}$
Therefore, when a power

Apply the laws of indices

In Grade 8, you learnt about the laws of indices. Remember, when simplifying problems, we do certain operations before others. You can use the acronym BODMAS to remind you of the order of operations: brackets, of, division, multiplication, addition and subtraction. The same order of operations also applies to working with indices.

There are additional rules (laws) for working with indices that are derived (worked out) from the definition. In this section, you will use these laws to solve problems.

Laws of indices

The laws of indices are derived from the definition of a power and they give short cuts for working with indices where the indices are positive, natural numbers.

Law 1: Multiplying powers that have the same base

$a^m \times a^n = a \times a \times \ldots$ (for m factors) $\times\, a \times a \times \ldots$ (for n factors)

$= a \times a \times \ldots$ (for $m + n$ factors)

$= a^{m+n}$

Therefore, if the bases are the same, add the indices.

Example: $2^5 \times 2^3 = (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^8$

$= 2^{5+3}$

Law 2: Dividing a power by another power that has the same base

$a^m \div a^n = \frac{a^m}{a^n} = \frac{a \times a \times \ldots \text{(for } m \text{ factors)}}{a \times a \times \ldots \text{(for } n \text{ factors)}}$

$= a \times a \times \ldots$ (for $m - n$ factors)

$= a^{m-n}$

Therefore, if the bases are the same, subtract the indices.

Example: $2^5 \div 2^3 = \frac{2^5}{2^3}$

$= \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2}$

$= \frac{\not{2} \times \not{2} \times \not{2} \times 2 \times 2}{\not{2} \times \not{2} \times \not{2}}$

$= 2^2$

$= 2^{5-3}$

Law 3: Raising a power to an index (also sometimes called double indices)

$(a^m)^n = (a^m) \times (a^m) \ldots$ (for n factors)

Example: $(a^2)^3 = (a^2) \times (a^2) \times (a^2)$

$= (a \times a) \times (a \times a) \times (a \times a)$

$= a^6$

$= a^{2 \times 3}$

Therefore, when a power is raised to an index, multiply the indices.

Law 4: Raising a product of factors to an index

$(a \times b)^m = a^m \times b^m$

Example:

$15^3 = (3 \times 5)^3$

$= (3 \times 5) \times (3 \times 5) \times (3 \times 5)$ (Apply the commutative property.)

$= 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$= 3^3 \times 5^3$

Therefore, when raising a product of factors to an index, the result is the same as raising each factor separately to the index.

Worked example 1

Use the laws of indices above to simplify the following.

1 $5a^6 \times 2a^4$

2 $\frac{a^{x+1}}{a^{x-1}}$

3 $(x^4)^5$

4 $(3x)^3$

Answers

1 $5a^6 \times 2a^4 = 10a^{6+4} = 10a^{10}$

2 $\frac{a^{x+1}}{a^{x-1}} = a^{x+1-(x-1)} = a^{x+1-x+1} = a^2$

3 $(x^4)^5 = x^{4 \times 5} = x^{20}$

4 $(3x)^3 = 3^3 \times x^3 = 27x^3$

Activity 2

1 Simplify.

a) 2×2^2 b) $2^5 \times 2^5$ c) $3^2 \times 2^2$

d) $5a^2 \times 2a^3$ e) $a^{2-x} \times a^{x+1}$ f) $6^{a+2} \times 6^{2-a}$

g) $5^{2m-3n} \times 5^{2n-2m}$ h) $2^{-x} \times 2^{x-2} \times 4$ i) $2 \times 3b^4 \times -2b^2$

2 Simplify.

a) $m^3 \div m^2$ b) $m^2 \div m^4$ c) $(xy)^3 \div xy$

d) $m^6 \div (m^2 \times m^3)$ e) $p \times p^3 \div p^4$ f) $a^{x+5} \div a^{x+2}$

3 Simplify each expression.

a) $(m^2)^2$ b) $(a^3)^4$ c) $(2^3)^2$

d) $(x^2y^3)^2$ e) $(m^2)^3$ f) 16^3

4 Simplify.

a) $(4x)^3$ b) $(2^{x+1} \times 3^x)^2$ c) $(ab)^3$

d) $(vw^2)^3 \div v^2w^3$ e) $(9a^2)^2$ f) $(a^3b^3)^3$

5 Write each of the following in index form as a product of prime factors.

a) 10^x b) 21^{2x} c) 49^3

d) 25^{x+1} e) 81^3 f) 35^5

6 Write with a single exponent (index): $(((10^2)^3)^4)^5$

Simplify expressio
zero indices

The definition of indices o
The second law for indices
applies when $m > n$, in oth
than the index of the deno
What happens in the follo
- $m = n$, when the index is
- $m < n$, when the index is

We will use the second law
negative.

Remember, when you divid
subtract the index of the d
the indices are positive wh

How do we apply the law w

When $m = n$, we get $\frac{a^m}{a^m} = 1$

Applying the second law,

But these two answers mea

Therefore, $a^0 = 1$, which m

When $m < n$, we get that m

$\frac{a^3}{a^5} = \frac{a \times a \times a}{a \times a \times a \times a \times a} =$

and applying the second la

$\frac{a^3}{a^5} = a^{-2}$

These answers means the s

$a^{-2} = \frac{1}{a^2}$.

So, a number raised to a ne
reciprocal of the number

Multiplicative inverse

The reciprocal of a number
For example, $\frac{1}{2}$ is the mul
The product of a number a
For example: $2 \times \frac{1}{2} = 1$. (1
So, we can write $\frac{1}{2}$ as 2^{-1}.
The multiplicative inverse

Simplify expressions with positive, negative and zero indices

The definition of indices on page 16 applies when the index is a positive number. The second law for indices, dividing a power by a power that has the same base, applies when $m > n$, in other words, when the index of the numerator is greater than the index of the denominator and both have the same base.

What happens in the following cases?

- $m = n$, when the index is 0
- $m < n$, when the index is negative

We will use the second law to investigate an index that is 0 and an index that is negative.

Remember, when you divide a power by another power that has the same base, subtract the index of the denominator from the index of the numerator, where the indices are positive whole numbers and $m > n$: $a^m \div a^n = a^{m-n}$.

How do we apply the law when $m = n$ and $m < n$?

When $m = n$, we get $\frac{a^m}{a^m} = 1$ (according to the property of numbers).

Applying the second law, $\frac{a^m}{a^m} = a^{m-m} = a^0$.

But these two answers mean the same thing.

Therefore, $a^0 = 1$, which means that any value raised to the power of 0 equals 1.

When $m < n$, we get that $m - n < 0$, for example:

$$\frac{a^3}{a^5} = \frac{a \times a \times a}{a \times a \times a \times a \times a} = \frac{1}{a \times a} = \frac{1}{a^2} \quad \text{(according to the definition)}$$

and applying the second law we get:

$$\frac{a^3}{a^5} = a^{-2}$$

These answers means the same thing, therefore,

$$a^{-2} = \frac{1}{a^2}.$$

So, a number raised to a negative power is the **reciprocal** of the number raised to its positive equivalent.

> **New word**
>
> **reciprocal:** the number by which a number is multiplied to give a product of 1 (example: $3 \times \frac{1}{3} = 1$)

Multiplicative inverse

The reciprocal of a number is the number's multiplicative inverse.

For example, $\frac{1}{2}$ is the multiplicative inverse of 2.

The product of a number and its multiplicative inverse is 1.

For example: $2 \times \frac{1}{2} = 1$. (1 is the multiplicative unit.)

So, we can write $\frac{1}{2}$ as 2^{-1}.

The multiplicative inverse of x is x^{-1}; so that $x \times x^{-1} = x^0 = 1$.

Worked example 2

1 Simplify $a^4 \div a^{-7}$.

2 Rewrite each number as an expression with a positive index.

a) a^{-1} b) 5^{-2} c) 2^{-3} d) 25^{-1}

Answers

1 $a^4 \div a^{-7} = a^4 \div \frac{1}{a^7}$

$= a^4 \times a^7$

$= a^{11}$

or, $a^4 \div a^{-7} = \frac{a^4}{a^{-7}}$

$= a^{4-(-7)}$

$= a^{11}$

2 a) $a^{-1} = \frac{1}{a}$ b) $5^{-2} = \frac{1}{5^2}$ c) $2^{-3} = \frac{1}{2^3}$ d) $25^{-1} = 5^{-2} = \frac{1}{5^2}$

Activity 3

1 Simplify.

a) $c^4 \div c^0$ b) $a^2 \div a^{-2}$ c) $p^{-3} \div p^{-4}$

d) $1 \div y^0$ e) $r^{-5} \div r^{-6}$ f) $ab^4 \div a(b)^{-4}$

2 Simplify each expression.

a) $3^0 \times 3^2$ b) $40^0 \times 2^{-3}$ c) $3^2 \times 3^0 \times 3^{-2}$

d) $10^0 \times 10 \div 10$ e) $10^0 \times 10 \times 10^{-1}$ f) $12^0 \div 12^{-1}$

3 Rewrite each expression with a positive index. (Example: $b^{-3} = \frac{1}{b^3}$)

a) a^{-2} b) 2^{-3}

Fractions with negative indices

We know that $a^{-m} = \frac{1}{a^m}$; for example, $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

Let's investigate the meaning of $\frac{1}{a^{-m}}$.

If we apply the second law of indices and the fact that $a^0 = 1$, we can write:

$$\frac{1}{a^{-m}} = \frac{a^0}{a^{-m}} = a^{0-(-m)} = a^m.$$

Another way of interpreting this:

$\frac{1}{a^{-m}} = \frac{1}{\frac{1}{a^m}}$ (Division by a fraction is the same as multiplying by its reciprocal.)

$= 1 \times \frac{a^m}{1}$

$= a^m$

Therefore, $\frac{1}{a^{-m}} = a^m$

Examples: $\frac{1}{2^{-3}} = 2^3$ and $\frac{1}{5^{-2}} = 5^2$

Simplify expressions that contain fractional indices

We know what it means when the powers of indices are integers (when they are positive or negative numbers and when an index is 0).

We need to find what it means when an index is a fraction. In other words, what is the meaning of the following powers: $a^{\frac{1}{2}}$, $a^{\frac{3}{4}}$ or $a^{\frac{p}{q}}$?

According to the third law of indices, when we raise a power to a power:

$$(a^{\frac{p}{q}})^q = a^p.$$

To get $a^{\frac{p}{q}}$ on the left side of the equation, we need to take the qth root on both sides of the equation:

$$a^{\frac{p}{q}} = \sqrt[q]{a^p}$$

In all these applications, the base (in this case a) is limited to positive values, $a > 0$.

Worked example 3

Simplify the following without using a calculator. Answers must have positive indices.

1 $8^{\frac{2}{3}}$ 2 $8^{\frac{-2}{3}}$ 3 $\dfrac{3^0+1}{4^{\frac{1}{2}}}$ 4 $\sqrt[4]{\dfrac{3^a \times 9^{a+1}}{27^{a+2}}}$

Answers

1 $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = \sqrt[3]{4^3} = 4$

or $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^{\frac{3\times2}{3}} = 2^2 = 4$

2 $8^{\frac{-2}{3}} = 8^{-1\times\frac{2}{3}} = (2^{-3})^{\frac{2}{3}} = 2^{-\frac{3\times2}{3}} = 2^{-2} = \frac{1}{4}$

3 $\dfrac{3^0+1}{4^{\frac{1}{2}}} = \dfrac{1+1}{\sqrt{4}} = \dfrac{2}{2} = 1$

4 $\sqrt[4]{\dfrac{3^a \times 9^{a+1}}{27^{a+2}}}$

$= \sqrt[4]{\dfrac{3^a \times 3^{2(a+1)}}{3^{3(a+2)}}}$

$= \sqrt[4]{\dfrac{3^a \times 3^{2a+2}}{3^{3a+6}}}$

$= \sqrt[4]{3^{a+2a+2-3a-6}}$

$= (3^{-4})^{\frac{1}{4}}$

$= 3^{-1}$

$= \frac{1}{3}$

Activity 4

1 Write the following as powers with positive indices.

a) $\sqrt[3]{a}$ b) $\sqrt{32}$ c) $\sqrt{9^{-1}}$

d) $\sqrt[3]{2}$ e) $\sqrt{4^{-3}}$ f) $\sqrt[3]{8^2}$

2 Rewrite each expression with positive indices.

a) $\frac{1}{2^{-1}}$ b) -2^{-3} c) $\left(\frac{1}{3}\right)^{-2}$

d) $\frac{3^{-2}}{3^2}$ e) $125^{-\frac{1}{3}}$ f) 7^{-2}

3 Simplify and leave your answers with positive indices.

a) $\frac{2^{n-4} \times 16^{1+n}}{8^{-n} \times 4^{-4n+1}}$ b) $(2^{-1} + 3^{-1})^2$ c) $\left(16^{\frac{1}{4}} + 32^{-\frac{2}{5}}\right)^{\frac{1}{2}}$

d) $\frac{7^{x+1}(4^x)^2}{14^{x-1} \times 2^{3x+1}}$ e) $\frac{2^n \times 8^{n+2}}{4^{3n} \times 2^{-2n}}$ f) $\frac{50^{x-2} \times 5 \times 20^{4-x}}{4^{-x} \times 10^{x+1}}$

g) $\frac{x^2 \times x^4}{x^3}$ h) $\frac{c^3 \times c}{c^2}$ i) $\frac{ab^2 \times a^2b}{ab}$

Solve equations involving indices

There are two types of equation that include indices:

- equations with an unknown index, such as $3^x = 81$
- equations with an unknown base, such as $x^5 = 32$.

Unknown index (or exponent)

When the index is the unknown value, solve the equation by writing both sides as a power of the same base. (Make the base on both sides of the equality sign the same.)

$3^x = 81$

$3^x = 3^4$; therefore $x = 4$

Worked example 5

Solve for x in each case.

1 $2^{x+1} = 64$ 2 $5^{x-1} = 125$

Answers

1 $2^{x+1} = 64$ Write 64 as a product of its prime factors in index form.

$2^{x+1} = 2^6$ Left side equals right side, bases are the same, so indices are the same.

$x + 1 = 6$

$x = 5$

2 $5^{x-1} = 125$

$5^{x-1} = 5^3$

$x - 1 = 3$

$x = 4$

Unknown base

When the base is the unk
sides of the equation.

$x^5 = 32$

$x^5 = 2^5$; therefore $x =$

You could use the followi
an appropriate power.

Example:

$x^5 = 32$

$(x^5)^{\frac{1}{5}} = (32)^{\frac{1}{5}}$

$x = (2^5)^{\frac{1}{5}}$

$x = 2$

Worked example 6

Solve for x.

1 $5x^{\frac{1}{2}} = 45$

Answers

1 $5x^{\frac{1}{2}} = 45$

$x^{\frac{1}{2}} = 9$

$(x^{\frac{1}{2}})^2 = 9^2$

$x = 81$

Activity 5

1 Solve the following

a) $2^x = 8$

d) $(3)(5^{x-1}) = 75$

g) $(9)(3^x) - 1 = 0$

2 Solve the following

a) $8x^{-\frac{2}{3}} = \frac{2}{9}$

d) $5(x + 1)^{\frac{1}{4}} = 10$

3 Solve for x: $\left(\frac{1}{2}\right)^{x^2 - 9} =$

Unknown base

When the base is the unknown value, try to make the index the same on both sides of the equation.

$x^5 = 32$

$x^5 = 2^5$; therefore $x = 2$

You could use the following technique to raise the side with the unknown base to an appropriate power.

Example:

$x^5 = 32$

$(x^5)^{\frac{1}{5}} = (32)^{\frac{1}{5}}$ $\quad (x^{\frac{5}{1} \times \frac{1}{5}} = x)$

$x = (2^5)^{\frac{1}{5}}$ $\quad (2^{\frac{5}{1} \times \frac{1}{5}} = 2)$

$x = 2$

Worked example 6

Solve for x.

1 $5x^{\frac{1}{2}} = 45$

2 $6x^{-\frac{3}{2}} - 162 = 0$

3 $(3x) \times \sqrt{x} = 81$

Answers

1 $5x^{\frac{1}{2}} = 45$

$x^{\frac{1}{2}} = 9$

$(x^{\frac{1}{2}})^2 = 9^2$

$x = 81$

2 $6x^{-\frac{3}{2}} - 162 = 0$

$6x^{-\frac{3}{2}} = 162$

$x^{-\frac{3}{2}} = 27$

$(x^{-\frac{3}{2}})^{-\frac{2}{3}} = 27^{-\frac{2}{3}}$

$x = (3^3)^{-\frac{2}{3}}$

$x = 3^{-2}$

$x = \frac{1}{9}$

3 $(3x) \times \sqrt{x} = 81$

$(3x)(x^{\frac{1}{2}}) = 3^4$

$x^{\frac{3}{2}} = \frac{3^4}{3}$

$(x^{\frac{3}{2}})^{\frac{2}{3}} = (3^3)^{\frac{2}{3}}$

$x = 3^2$

$= 9$

Activity 5

1 Solve the following equations for x.

a) $2^x = 8$ b) $9^x = 27$ c) $2^{2x+6} = 16^{-x}$

d) $(3)(5^{x-1}) = 75$ e) $2^{x-1} = \frac{1}{8}$ f) $3^{2x} = \frac{1}{81}$

g) $(9)(3^x) - 1 = 0$ h) $\sqrt[x]{81} = 27$ i) $(4)(5^{x+1}) = 100$

2 Solve the following equations for x.

a) $8x^{-\frac{2}{3}} = \frac{2}{9}$ b) $x^{\frac{2}{3}} = \frac{9}{4}$ c) $x^{-\frac{2}{3}} = (16^{-\frac{1}{6}})(9^{\frac{1}{3}})$

d) $5(x+1)^{\frac{1}{4}} = 10$ e) $(x^{\frac{1}{2}} + 1)^{\frac{1}{2}} = 2$ f) $(2x)(\sqrt{x}) = 16$

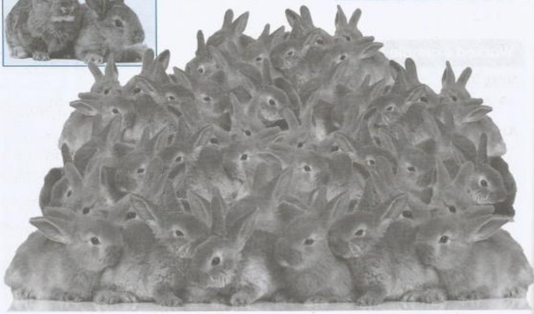
3 Solve for x: $\left(\frac{1}{2}\right)^{x^2-9} = 4^{x+3}$

Solve problems that involve applying rules of indices

We use exponential equations to solve exponential growth problems. (The growth of a population is an example of an exponential growth problem.) Exponential growth is described as growth that becomes increasingly faster. For example, a rabbit population that breed **unhindered** will grow from two rabbits to 2^n rabbits within n generations, if each pair of rabbits has only one pair of rabbits as offspring.

New word

unhindered: without being stopped



Use your knowledge of indices and solving equations to do the following activity.

Activity 6

1 The number of ancestors, y in the nth generation before you is given by $y = 2^n$. Write down how many ancestors you have in each generation before you.
   a) the seventh generation
   b) the tenth generation

2 A boy worked out how many ancestors he had in certain generations before him. Calculate the generation in which he had the following number of ancestors.
   a) 4 096 ancestors
   b) 16 384 ancestors

Activity 6 (continue

3 In an election a poli
received 5^9 votes. Ho
voted for him?

4 A planet has a diame
an index.

5 The volume of a cub
length of the cube.
a) Calculate the vol
b) Calculate the side

6 A certain bacterium
number of cells in a

a) Write down the n
b) How many cells
c) Which generatio
d) Which generatio

of indices

...lems. (The growth ...) Exponential ...or example, a ...bbits to 2^n rabbits ...rabbits as

...ew word

... without being

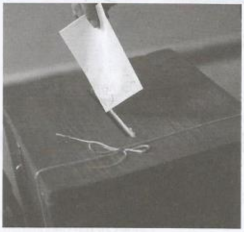
...following activity.

...is given by $y = 2^n$. ...on before you.

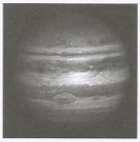
...enerations before ...g number of

Activity 6 (continued)

3 In an election a politician received 5^9 votes. How many people voted for him?



4 A planet has a diameter of 9^4 km. Give the diameter as a number without an index.



5 The volume of a cube is given by $V = s^3$, where s is the side length of the cube.

a) Calculate the volume of a cube with a side length of 5 cm.
b) Calculate the side length of a cube with a volume of 343 cm^3.

6 A certain bacterium grows by making three cells from every one cell. The number of cells in a generation is given by $y = 3^x$.

Bacteria

a) Write down the number of cells in the 9th generation.
b) How many cells will there be in the 11th generation?
c) Which generation gave 531 441 cells?
d) Which generation gives $(3^5)(3^6)$ cells? How many cells are there?

TOPIC
2 Summary, revision and assessment

Summary

- Definition: $a^n = a \times a \times a \times ...$ for n factors, where n is a positive whole number.
- The laws of indices apply for positive whole numbers, m and n.

Laws of indices

	Law	Description	Examples
1	$a^m \times a^n$ $= a^{m+n}$	When multiplying indices that have the same base, add the indices.	• $2^3 \times 2^4 = 2^{3+4} = 2^7 = 128$ • $a^5 \times a^2 = a^{5+2} = a^7$
2	$\frac{a^m}{a^n}$ $= a^{m-n}$	When dividing with indices that have the same base, subtract the index of the denominator from the index of the numerator.	• $\frac{3^3}{3^2} = 3^{3-2} = 3^1 = 3$ • $\frac{a^6}{a^3} = a^{6-3} = a^3$
3	$(a^m)^n$ $= a^{m \times n}$	When raising a power to an index, multiply the indices. Also called double indices.	$(3^3)^2 = (3^3)(3^3)$ $= 27 \times 27$ $= 729$ or, $(3^3)^2 = 3^{3 \times 2}$ $= 3^6$ $= 729$
4	$(a \times b)^m$ $= a^m \times b^m$	When raising a product to a power, the factors of the product can be raised to the power separately.	$(2 \times 7)^m = 2^m \times 7^m$ also $\left(\frac{3}{4}\right)^m = \frac{3^m}{5^m}$

- The meaning of a^n where the index is 0; a negative number or a fraction:
  - » $a^0 = 1$
  - » $a^{-n} = \frac{1}{a^n}$ (This can also be seen as the multiplicative inverse.)
  - » $\frac{1}{a^{-n}} = a^n$ (These are fractions with negative indices.)
  - » $a^{\frac{n}{m}} = \sqrt[m]{a^n}$

Solving equations that involve indices

In an equation, the left side equals the right side (=).

- In equations where the index is the unknown, make the bases on both sides the same and compare the indices on the right with those on the left.
- In equations where the unknown is in a base, make the indices on both sides the same and compare the base on the right with the base on the left, or raise the unknown power to the power that will make the index 1 and then do the same on the other side.
- You can use roots to solve many equations with fractional indices.

Revision exercis

1 Rewrite each expressio
a) $4a^{-1}$

2 Use the laws of indice
a) $(p^{-2})^{-1}$
d) $y^0 \div y^{-1}$
g) $(xy)^{-7} \div (xy)^8 \times (xy)^2$

3 Write the following in
a) $3^{\frac{1}{2}}$
d) $6^{\frac{3}{4}}$

4 Write the following in
a) $\sqrt[3]{a^2}$
d) $\sqrt[4]{a^3}$

5 Simplify.
a) $\sqrt[3]{8}$
d) $\sqrt[3]{64}$

6 Solve the following eq
a) $2^x = 2$
d) $3^{2x+3} = 3^x$

Assessment exe

1 Simplify.
a) $\frac{x^3}{y^2} \times \frac{y^2}{x^2}$
d) $\frac{12ab}{5a^2b^2} \times \frac{10a^3b^3}{3b}$

2 Simplify.
a) $\sqrt[3]{2^9}$
d) $\sqrt[3]{216}$

3 Solve the following eq
a) $2^{2x} = 64$
c) $2^{2x-1} = \frac{1}{8}$
e) $5 \times 3^{2x+2} = 135 \times 3$

4 If $x = -1$ and $y = 2$, cal

Revision exercises

1 Rewrite each expression with a positive index.

a) $4a^{-1}$ b) $\frac{3}{r^{-3}}$ c) b^{-3}

2 Use the laws of indices to simplify each expression.

a) $(p^{-2})^{-1}$ b) $(a^2b)^2$ c) $y^{-5} \div y^{-1}$
d) $y^0 \div y^{-1}$ e) $a^3 \times a^2 \div a$ f) $t^6 \div t^2 \times t^2$
g) $(xy)^{-7} \div (xy)^8 \times (xy)^2$ h) $\frac{12a^3b^2}{4a^2b}$ i) $\frac{5a^3 \times 3a^2b^3}{6a^4b^4}$

3 Write the following in the form $\sqrt[n]{a^m}$, where m and n are positive integers.

a) $3^{\frac{1}{2}}$ b) $5^{\frac{2}{3}}$ c) $8^{\frac{4}{3}}$
d) $6^{\frac{3}{4}}$ e) $2^{-\frac{1}{2}}$ f) $3^{-1\frac{1}{2}}$

4 Write the following in the form a^y where y is a whole number or a fraction.

a) $\sqrt[3]{a^2}$ b) $\sqrt[3]{a^4}$ c) $\sqrt[5]{a^3}$
d) $\sqrt[4]{a^3}$ e) $\frac{1}{\sqrt{a}}$ f) $\frac{1}{\sqrt[3]{a^5}}$

5 Simplify.

a) $\sqrt[3]{8}$ b) $\sqrt[3]{27}$ c) $\sqrt[3]{2^6}$
d) $\sqrt[3]{64}$ e) $\sqrt[4]{2^8}$ f) $\sqrt{a^6} \div \sqrt[3]{a^9}$

6 Solve the following equations.

a) $2^x = 2$ b) $3^{2x} = 3^4$ c) $2^{x+1} = 2^2$
d) $3^{2x+3} = 3^x$ e) $5^x = 1$ f) $3^{x+2} = 27$

Assessment exercises

1 Simplify.

a) $\frac{x^3}{y^2} \times \frac{y^2}{x^2}$ b) $\frac{2ab}{a^2} \times \frac{3ab}{b^2}$ c) $\frac{15xy^3}{4x^2y} \times \frac{8x^2y}{5x^2y^2}$
d) $\frac{12ab}{5a^2b^2} \times \frac{10a^3b^3}{3b}$ e) $\frac{2a^2 \times 3a^3}{a}$ f) $\frac{45x^3y \times 9x^3y^2}{81x^2y^2 \times 10x^3y^2}$

2 Simplify.

a) $\sqrt[3]{2^9}$ b) $\sqrt[3]{27^2}$ c) $\sqrt[3]{1{,}000}$
d) $\sqrt[3]{216}$ e) $\sqrt[3]{3^{12}}$ f) $\sqrt{3^6}$

3 Solve the following equations.

a) $2^{2x} = 64$ b) $3^4 = \frac{1}{3x}$
c) $2^{2x-1} = \frac{1}{8}$ d) $5^{2x} = \frac{1}{125}$
e) $5 \times 3^{2x+2} = 135 \times 3^{x+1}$ f) $5^x \times 6 = 150$

4 If $x = -1$ and $y = 2$, calculate the value of: $(4y^2)^2 - 18(3)^{xy} - y^{2y}$.

TOPIC
3 Algebra

Sub-topic	Specific Outcomes
Basic processes	• Expand and simplify expressions. • Factorise algebraic expressions. • Simplify algebraic fractions.

Starter activity

1 Simplify each problem as far as possible.
   a) 2 tennis balls + 1 football ball + 5 footballs + 3 tennis balls
   b) 7 football balls + 6 tennis balls – 4 football balls
   c) $9p + 12p - 3p + 14p$
   d) $2a + 3a + 4p - 2p$
   e) $6m + 2n - 4m - 3n + 8m$
   f) $25 + 13a - 11a - 10 + 3a^2$
2 Explain to a partner how you simplified the problems in question 1.

SUB-TOPIC 1

Basic processes

Introduction

Algebra is the part of mathematics in which letters are used to represent numbers and quantities.

An algebraic expression is a mathematical phrase that can contain numbers, operators (such as addition, subtraction, multiplication and division) and at least one **variable** (such as a, b, x and y).

Examples of algebraic expressions:

$3a$	$2b + 12$
$a - 7b$	$15 \times 12x$
$x \div 2$	$\frac{4x}{y}$

New word

variable: symbol for a number or for a few different numbers

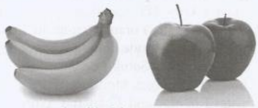
Simplifying expressions

To simplify an expression means to make it as simple as possible. We simplify expressions because it is easier to substitute numbers for variables into an expression *after* it has been simplified.

Example: $4a + 2ab + 4ab - 2a - 2ab + ab$ can be simplified to $2a + 5ab$

The starter activity shows that you can simplify an expression that contains like terms (such as ab in the above example). You cannot simplify an expression that only contains unlike terms (such as the expressions $2a + 3b$ or 3 bananas and 2 apples).

We usually group like terms when we simplify an expression.



You cannot simplify 3 bananas and 2 apples!

New words

like terms: the same variables (letters)
unlike terms: different variables

Worked example 1

1 Simplify each expression as far as possible.
   a) $5x + 8x$ b) $3m - 2m$
   c) $4x + 3y$ d) $5a + 3b - 2a + 4b$

2 Mary and Zisa went to a shop to buy pens and crayons. Mary bought 5 pens and 7 crayons. Zisa bought 8 pens and 9 crayons. How many pens and crayons did they buy altogether?

Mary bought 5 pens and 7 crayons.

Worked example 1 (continued)

Answers

1 a) $5x + 8x = 13x$

b) $3m - 2m = m$

c) You cannot add $4x$ and $3y$ because they are unlike terms.

d) $(5a - 2a) + (3b + 4b) = 3a + 7b$

2 Let p represent pens, and c represent crayons.

Mary: $5p + 7c$

Zisa: $8p + 9c$

Total number of pens and crayons

$= (5p + 8p) + (7c + 9c)$

$= 13p + 16c$

The two girls bought 13 pens and 16 crayons in total.

Activity 1

1 Simplify each expression.

a) $3p + 2q + 5p$
b) $2x + 5x + 3y + 4y$
c) $3j - 8r - 3j$
d) $l - k + 4l - 5k$
e) $7m - 3n - 5m + 6n$
f) $7d - 6e + 12e - 10d$
g) $4b + 11a - 4a - 11b$
h) $3pq + 2pq - 5pq$
i) $xy + 4xy + 3xz$
j) $2mn + 4jk - 4mn$

2 Sombo bought 4 oranges from the fruit market, and Monde bought 3 oranges and 5 bananas. Sombo gave her brother 2 oranges, Monde gave his sister 1 orange and 2 bananas. Write down an expression for how many oranges and bananas Sombo and Monde have left in total.

Monde bought 3 oranges and 5 bananas.

Expanding expressions

When an expression contains brackets, you have to expand the expression before you can simplify it.

Examples of expressions that contain brackets:

$3(a)$ $\quad$ $2(a + b)$

$5x(1 - 4y)$ $\quad$ $(a + b)(c + d)$

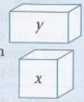
To expand an expression means to multiply everything in front of a pair of brackets with everything inside the brackets. When you have expanded an expression, it will not contain any brackets and it will be easier to simplify it.

Worked example 2

1 Expand each expression.

a) $x(y + z)$ b) $xy(x - y^2)$
c) $2(2y - 5x) + 3(y - x)$ d) $(x + 1)(x - 3)$
e) $(x - 2)^2$

2 Nzala and Lukundo need to carry boxes of books into a classroom. There are two types of box: one containing x books and one containing y books. Each man can carry one box of each kind on each trip. After Nzala has made 8 trips and Lukundo has made 11 trips, find the following (in terms of x and y):

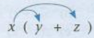


a) the number of book each man has taken into the classroom
b) the total number of books that have been taken into the classroom.

Answers

1 Multiply the term outside the brackets by each term inside the brackets.

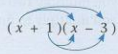
a) $x(y + z)$
$= xy + xz$



b) $xy(x - y^2)$
$= xy(x) - xy(y^2)$
$= x^2y - xy^3$

c) $2(2y - 5x) + 3(y - x)$
$= 4y - 10x + 3y - 3x$
$= (4y + 3y) + (-10x - 3x)$ Group like terms.
$= 7y - 13x$

d) Multiply each term in the first bracket by each term in the second bracket.
$(x + 1)(x - 3)$
$= x(x - 3) + 1(x - 3)$
$= x^2 - 3x + x - 3$
$= x^2 - 2x - 3$



e) $(x - 2)^2$
$= (x - 2)(x - 2)$
$= x(x - 2) - 2(x - 2)$
$= x^2 - 2x - 2x + 4$
$= x^2 - 4x + 4$

2 a) Each man carries two boxes on each trip, i.e. $x + y$ books.
Nzala carried: $8x + 8y = 8(x + y)$ books
Lukundo carried: $11x + 11y = 11(x + y)$ books

b) Total number of books taken into the classroom:
$8x + 11x + 8y + 11y$
$= 19x + 19y$
$= 19(x + y)$

Monde bought
anges and 5 bananas.

the expression before

unt of a pair of
e expanded an
sier to simplify it.

Activity 2

1 Expand each expression.
a) $2(p + 1)$ b) $3(x - y)$ c) $5j(k - 1)$ d) $z(3z + 4)$
e) $m(9 - m)$ f) $2r(5r + 3)$ g) $-3a(3 - a)$ h) $xy(8 + y)$
i) $pq(-q^2 - 3p^2)$ j) $2n(8m - 4n)$

2 Teza bought a box with 12 pens and 4 pencils. He gave 4 pens and 1 pencil away. On each of the next three days, he again bought a box with 12 pens and 4 pencils and gave away 4 pens and 1 pencil from each box. How many pens and pencils did he keep in total?

In the next activity, you will need to expand each expression and then simplify it.

Activity 3

1 Expand and simplify.
a) $2(k + 3) + 3(k + 2)$ b) $5(r + s) + (r - s)$ c) $(m - 4) - 3(m + 3)$
d) $4(y + 2) - (y - 8)$ e) $z(z - 2) - z(z - 4)$ f) $ab(a + b) - ab(a - b)$

2 Expand and simplify.
a) $(a + 1)(a + 7)$ b) $(k - 2)(k + 2)$ c) $(3f + 2)(f + 1)$
d) $(2n - 3)(5n - 3)$ e) $(j - 2k)(5j - 5k)$ f) $(6x - 3y)(x - 5y)$
g) $(e - 2)(e + 1) - e(e + 7)$ h) $(3b + 5)(3b - 5) - 2b(7b - 3)$

Factorising algebraic expressions

Factorising an algebraic expression means finding the factors of the expression.

As you know, when you multiply two or more numbers, the answer is called the product. The numbers you multiply to find a product are called the factors of the product.

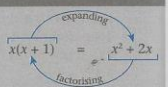
Examples

In $2 \times 3 \times 5 = 30$:
- 30 is the product of 2, 3 and 5
- 2, 3 and 5 are factors of 30.

In $x(x + 2) = x^2 + 2$:
- x and $(x + 2)$ are algebraic factors of $x^2 + 2$.

Note

Factorising is the opposite of expanding:
- When expanding an expression, you remove the brackets
- When factorising an expression, you find the factors. Sometimes, you use brackets to write the factors of an expression.



Factorising the expression $a + ab$ means finding algebraic numbers that will give $a + ab$ as the product when you multiply them with each other.

For example, $a + ab = a(1 + b)$, and so the factors of $a + ab$ are a and $1 + b$.

Methods we generally use to factorise algebraic expressions include:

- finding common factors
- grouping terms
- finding factors of quadratic expressions
- finding the difference of two squares.

Finding common factors

When factorising, we look for numbers or letters that are common to the terms in an expression.

A factor is a number that divides into another number exactly without leaving a remainder. For example, 3 is a factor of 6, but it is not a factor of 7.

A common factor is a number that divides exactly into (two or more) numbers. To find the common factors of two numbers, we write down all the factors of both numbers and then look for the factors that are common to both numbers.

In algebra, common factors often include variables.

Worked example 3

Factorise using common factors.

1 $2p + 4$ 2 $r + 4r^2$ 3 $9xy - 3x - 3x^2$

Answers

1 $2p + 4 = 2(p + 2)$ The factor 2 is common to both terms.
2 $r + 4r^2 = r(1 + 4r)$
3 $9xy - 3x - 3x^2 = 3x(3y - 1 - x)$

Activity 4

Factorise each expression.

1 $3z + 9$
2 $k - jk$
3 $3 - 12e$
4 $xy - xz$
5 $12r^2 + 8r$
6 $15n^2 - 25n$
7 $24e^2 + 6e^4$
8 $pqr + qr^2$
9 $6 - 12z - 18y$
10 $64x - 56xy - 48x^2$
11 $7b + 7b^2c - b^3c^3$
12 $uv^2 + u^3v^2 - uv^3$

Grouping terms

Sometimes we can group expressions that have a few terms so that each group contains a common factor. To do this, start by rearranging the terms if necessary so that you can place terms with common factors in groups.

Worked example 4

Factorise using the grouping of terms.

1 $ax + bx + ay + by$ 2 $h^2 + hp - ah - ap$ 3 $ac - bx + bc - ax$

Answers

1 $ax + bx + ay + by$
$= ax + bx + ay + by$ Group terms in pairs.
$= x(a + b) + y(a + b)$
$= (a + b)(x + y)$

2 $h^2 + hp - ah - ap$
$= h(h + p) - a(h + p)$
$= (h + p)(h - a)$

3 $ac - bx + bc - ax$
$= ac - ax + bc - bx$ Rearrange the terms.
$= a(c - x) + b(c - x)$
$= (c - x)(a + b)$

Activity 5

Factorise using the grouping of terms.

1 $pr + ps + qr + qs$
2 $5j + ij + 5k + ik$
3 $am + an + bm + bn$
4 $rs - rt + 3s - 3t$
5 $6x + xy - 6u - uy$
6 $ab + 2a + 2b + 4$
7 $2x + xy - 6 - 3y$
8 $fk - fg + k^2 - gk$
9 $3uv^2 - 4uv - 9v + 12$
10 $az + 5a - 2z - 10$
11 $k^2 + ks - 9k - 9s$
12 $6 - 3p - 4q + 2pq$

Factors of quadratic expressions

A **quadratic** expression is in the form $ax^2 + bx + c$, where a, b and c are numbers and $a \neq 0$. One or more of the terms is squared. For example, $2x^2 + 5x - 9$ and $j^2 - 4jk + 3k^2$ are quadratic expressions.

New word

quadratic: an expression in the form $ax^2 + bx + c$

To factorise a quadratic expression, follow these steps:

Step 1 Form a product and a sum. The product is given by $a \times c$ and the sum is b.
Step 2 Find two numbers with a product that is equal to $a \times c$ and a sum that is equal to b.
Step 3 Split b into a sum of two parts to change the number of terms in the original quadratic expression from three to four.
Step 4 Pair the terms and find a common factor.
Step 5 Factorise using grouping.

Worked example 5

Factorise.
1 $x^2 + 3x + 2$

Answers

1 $x^2 + 3x + 2$
$ax^2 + bx + c$
Step 1 Product:
Step 2 Two num
Step 3 Split 3: 1 +
Step 4 $x^2 + 3x +$
Step 5 $x^2 + 3x +$

2 $2x^2 + x - 6$
$ax^2 + bx - 6$
Step 1 Product:
Step 2 Two num
Step 3 $2x^2 + x -$
Step 4 $2x^2 + x -$
Step 5 $x(2x - 3)$

Activity 6

1 Factorise each exp
a) $x^2 + 6x + 5$
e) $x^2 - 8x + 16$
2 Factorise.
a) $2k^2 - 4k - 6$
e) $6 - 5f - 4f^2$
3 Each quadratic exp
quadratic expressio
a) $a^2 + 2ab + b^2$
c) $r^2 - 6rs + 9s^2$

The difference

The expression $a^2 - b^2$
between the square of
factors of the form $(a +$
$a^2 - b^2 = (a + b)(a$
You can use the differe

Worked example 5

Factorise.

1 $x^2 + 3x + 2$ 2 $2x^2 + x - 6$

Answers

1 $x^2 + 3x + 2$

$ax^2 + bx + c$ coefficients: $a = 1$, $b = 3$ and $c = 2$

Step 1 Product: $a \times c = 1 \times 2 = 2$; sum: $b = 3$

Step 2 Two numbers with a product of 2 and a sum of 3: 1 and 2

Step 3 Split 3: 1 + 2

Step 4 $x^2 + 3x + 2 = x^2 + x + 2x + 2$

Step 5 $x^2 + 3x + 2 = x(x + 1) + 2(x + 1)$

$= (x + 1)(x + 2)$

2 $2x^2 + x - 6$

$ax^2 + bx - 6$ coefficients: $a = 2$, $b = 1$ and $c = -6$

Step 1 Product: $a \times c = -12$; sum: $b = 1$

Step 2 Two numbers with a product of −12 and a sum of 1: 4 and −3

Step 3 $2x^2 + x - 6 = 2x^2 - 3x + 4x - 6$

Step 4 $2x^2 + x - 6 = x(2x - 3) + 2(2x - 3)$

Step 5 $x(2x - 3) + 2(2x - 3) = (2x - 3)(x + 2)$

Activity 6

1 Factorise each expression.

a) $x^2 + 6x + 5$ b) $x^2 + 8x + 7$ c) $x^2 + 6x + 8$ d) $20 + 9x + x^2$

e) $x^2 - 8x + 16$ f) $x^2 + x - 2$ g) $2 + x - x^2$ h) $30 - x - x^2$

2 Factorise.

a) $2k^2 - 4k - 6$ b) $3m^2 - 2m - 5$ c) $5h^2 + 16h + 3$ d) $9x^2 + 3x - 2$

e) $6 - 5f - 4f^2$ f) $5r^2 + 13r + 6$ g) $6n^2 + 11n + 3$ h) $2s^2 + 7s - 30$

3 Each quadratic expression has two factors that are the same. Factorise the quadratic expressions.

a) $a^2 + 2ab + b^2$ b) $j^2 - 8jk + 16k^2$

c) $r^2 - 6rs + 9s^2$ d) $25y^2 - 20xy + 4x^2$

The difference of two squares

The expression $a^2 - b^2$ is called a difference of two squares. It is the difference between the square of a and the square of b. All expressions of this form have factors of the form $(a + b)$ and $(a - b)$.

$$a^2 - b^2 = (a + b)(a - b)$$

You can use the difference of two squares to simplify numerical expressions.

The worked examples will help you understand how the difference of two squares works.

Worked example 6

Simplify by using the difference of two squares.

1 $8^2 - 6^2$

2 $100^2 - 90^2$

Answers

1 $8^2 - 6^2 = (8 + 6)(8 - 6)$
$= 14 \times 2$
$= 28$

2 $100^2 - 90^2 = (100 + 90)(100 - 90)$
$= 190 \times 10$
$= 1\ 900$

The difference of two squares in algebraic expressions

Now you can apply what you have learnt to algebraic expressions. Start by working through the worked examples.

Worked example 7

Factorise algebraic expressions using the difference of two squares.

1 $p^2 - 4^2$

2 $a^2 - 9$

3 $2x^2 - 50$

4 $18m^2 - 72n^2$

Answers

1 $p^2 - 4^2 = (p + 2)(p - 2)$

2 Rewrite $a^2 - 9$ as $a^2 - 3^2$.
$a^2 - 3^2 = (a + 3)(a - 3)$

3 Simplify: $2x^2 - 50 = 2(x^2 - 25)$
Rewrite $2(x^2 - 25)$ as $2(x^2 - 5^2)$.
$2(x^2 - 5^2) = 2(x + 5)(x - 5)$

4 This expression does not seem to have factors of the form $(a + b)$ and $(a - b)$, but 18 is a common factor to both $18m^2$ and $72n^2$.
$18m^2 - 72n^2 = 18(m^2 - 4n^2)$
$= 18[m^2 - (2n)^2]$
$= 18(m + 2n)(m - 2n)$

Activity 7

1 Factorise completely.

a) $a^2 - 5^2$ b) $p^2 - q^2$ c) $x^2 - 3^2$ d) $9 - r^2$

e) $f^2 - 16$ f) $100 - m^2$ g) $36 - y^2$ h) $x^2 - 49$

i) $5x^2 - 20$ j) $4 - 36p^2$ k) $4a^2 - 9b^2$ l) $(1 + i)^2 - i^2$

2 Simplify each expression.

a) $12^2 - 2^2$ b) $18^2 - 8^2$ c) $24^2 - 14^2$ d) $105^2 - 5^2$

Activity 7 (contiued)

3 Simplify $\frac{4u^2 - 16v^2}{4u + 8v}$.

4 Factorise $a^2 - b^2$. Given that $a^2 - b^2 = P$ and $P = 56$ when $a - b = 4$, find the value of $a + b$.

The worked examples that follow show you how to expand more expressions with two terms.

Worked example 8

1 Expand: $(2x - 3)^2$

2 Expand and simplify: $(x - 3)^2 + (2 - x)^2$

Answers

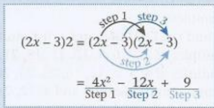
1 Follow the steps:

Step 1 $(2x)^2 = 4x^2$

Step 2 $2(2x) \times (-3)) = -12x$

Step 3 $(-3)^2 = 9$

Answer: $4x^2 - 12x + 9$



2 $(x - 3)^2 + (2 - x)^2$

$= x^2 + 2(-3x) + 9 + 4 + 2(-2x) + x^2$

$= x^2 - 6x + 9 + 4 - 4x + x^2$

$= (x^2 + x^2) + (-6x - 4x) + (9 + 4)$ Group like terms in brackets.

$= 2x^2 - 10x + 13$

Activity 8

1 Expand each expression.

a) $(x + 1)^2$ b) $(k - 2)^2$ c) $(3p + 1)^2$

d) $(2s - 3t)^2$ e) $(2a - 4b)^2$ f) $(5 + ab)^2$

2 Expand and simplify.

a) $(3 + i)^2 + (2 - i)^2$ b) $(r + 3)^2 + (r - 3)^2$ c) $(m + n)^2 + (m + n)^2$

d) $(z - 1)^2 + (1 - z)^2$ e) $(k + 4)^2 - (k - 4)^2$ f) $(3f + g)^2 - (3f - g)^2$

Simplify algebraic fractions

As you know, algebraic fractions contain variables. In this section, you will use addition, subtraction, multiplication and division to simplify algebraic fractions.

Add and subtract with algebraic fractions

Use the same principles you used to simplify algebraic expressions when you have to add and subtract with algebraic fractions.

Remember:

- When the **denominators** are the same, add the **numerators**.
- When the denominators differ, find the lowest common multiple (LCM) of the denominators. Convert all fractions so that they have the same LCM. Simplify the numerators. You can find the LCM by multiplying the terms.

New words

denominator: number under the line in a fraction

numerator: number above the line in a fraction

Lowest common multiple

The lowest common multiple (LCM) of two or more numbers is the smallest (lowest in value) number in the list of multiples of the numbers. The LCM is divisible exactly by each term without leaving a remainder. You can use the LCM to find the best denominator for all the fractions in an expression when you need to simplify it using addition and subtraction.

Examples:

To find the LCM of 3 and 4, list multiples of both numbers:

multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, ..

multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, ..

common multiples of 3 and 4: 12, 24, 36

LCM of 3 and 4: 12

To find the LCM of $3a$ and $4b$, use the LCM of coefficients 3 and 4 (which is 12) and the product of a and b (which is ab).

LCM of $3a$ and $4b$: $12ab$

Worked example 9

Simplify each expression. Start by finding the LCM of the denominators.

1 $\frac{1}{a} + \frac{2}{3a}$ 2 $\frac{2}{ab} + \frac{a}{b} - \frac{b}{a}$

Answers

1 $\frac{1}{a} + \frac{2}{3a}$ The LCM of a and $3a$ is $3a$.

$= \frac{3}{3a} + \frac{2}{3a}$ Check: $\frac{3}{3a} = \frac{1}{a}$

$= \frac{3+2}{3a}$

$= \frac{5}{3a}$

2 $\frac{2}{ab} + \frac{a}{b} - \frac{b}{a}$ The LCM of a, b and ab is ab.

$= \frac{2}{ab} + \frac{a^2}{ab} - \frac{b^2}{ab}$ Check: $\frac{a^2}{ab} = \frac{a}{b}$ and $\frac{b^2}{ab} = \frac{b}{a}$

$= \frac{2 + a^2 - b^2}{ab}$

Activity 9

1 Find LCM of each se

a) 40 and 50

c) $4a$, $5a^2$ and 2

2 Simplify each expre

a) $\frac{12x}{y} - \frac{2}{y^2}$ b

c) $\frac{4p}{3q} - \frac{4p}{q^2}$ d

Worked example 1

1 Simplify.

a) $\frac{1}{m} + \frac{2}{m}$

2 Express $\frac{3}{x-2} +$

Answers

1 a) $\frac{1}{m} + \frac{2}{m}$ Th

$= \frac{1+2}{m}$

$= \frac{3}{m}$

b) $\frac{3}{b} - \frac{2}{b}$

$= \frac{3-2}{b}$

$= \frac{1}{b}$

c) $2 + \frac{4}{y} - \frac{3}{2y}$

$= \frac{2}{1} + \frac{4}{y} - \frac{3}{2y}$

$= \frac{4y+8-3}{2y}$

$= \frac{4y+5}{2y}$

2 The LCM of $(x-2$

$\frac{3}{x-2} + \frac{1}{x-3} =$

ew words

umber under the line in a

ber above the line in a

ou can find the LCM

s is the smallest
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ou can use the LCM
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and 4 (which is 12)

enominators.

Activity 9

1 Find LCM of each set of numbers.

a) 40 and 50

b) p and p^2

c) $4a$, $5a^2$ and 2

d) $5pq$, $3p$ and $4q^2$

2 Simplify each expression.

a) $\frac{12x}{y} - \frac{2}{y^2}$

b) $\frac{4m}{3n} + \frac{5n}{2m}$

c) $\frac{4p}{3q} - \frac{4p}{q^2}$

d) $\frac{7x}{x^2} - \frac{4x^2}{x^3}$

Hint

Simplify as far as possible before you find the LCM of the denominators!

Worked example 10

1 Simplify.

a) $\frac{1}{m} + \frac{2}{m}$

b) $\frac{3}{b} - \frac{2}{b}$

c) $2 + \frac{4}{y} - \frac{3}{2y}$

2 Express $\frac{3}{x-2} + \frac{1}{x-3}$ as a single fraction.

Answers

1 a) $\frac{1}{m} + \frac{2}{m}$ The denominators are the same, so add the numerators.

$= \frac{1+2}{m}$

$= \frac{3}{m}$

b) $\frac{3}{b} - \frac{2}{b}$

$= \frac{3-2}{b}$

$= \frac{1}{b}$

c) $2 + \frac{4}{y} - \frac{3}{2y}$ Treat 2 as the fraction $\frac{2}{1}$.

$= \frac{2}{1} + \frac{4}{y} - \frac{3}{2y}$ The LCM of 1, y and $2y$ is $2y$.

$= \frac{4y+8-3}{2y}$

$= \frac{4y+5}{2y}$

2 The LCM of $(x-2)$ and $(x-3)$ is $(x-2)(x-3)$.

$$\frac{3}{x-2} + \frac{1}{x-3} = \frac{3(x-3)+1(x-2)}{(x-2)(x-3)}$$

$$= \frac{3x-9+x-2}{(x-2)(x-3)}$$

$$= \frac{4x-11}{(x-2)(x-3)}$$

Note

Multiply:

- 3 by the term $(x-3)$
- the term $(x-2)$ by 1
- $(x-2)$ by $(x-3)$.

$\frac{3}{x-2} + \frac{1}{x-3}$

Activity 10

1 Simplify.

a) $\frac{a}{3} + \frac{a}{4}$ b) $\frac{3y}{5} + \frac{y}{2}$ c) $k - \frac{k}{4} + \frac{2k}{3}$ d) $\frac{5}{b} - \frac{4}{b}$

e) $\frac{b}{c} + \frac{b}{c}$ f) $\frac{x+2}{4} + \frac{x+1}{3}$ g) $1 - \frac{x-5}{6}$ h) $\frac{2e-3}{5} + 2e$

i) $\frac{5m+4}{3} - \frac{m+5}{4}$ j) $3c^3\left(\frac{1}{c^3} - c^2\right)$

2 Write each expression as a single fraction in its lowest terms.

a) $\frac{1}{u+2} + \frac{1}{u+1}$ b) $\frac{3}{x+5} - \frac{2}{x+3}$ c) $\frac{5}{2-m} - \frac{3}{2+m}$ d) $\frac{4}{y-5} - \frac{2}{3y+7}$

e) $\frac{8}{2-b} - \frac{1}{4-b}$ f) $\frac{5}{2e+5} + \frac{7}{e-3}$ g) $\frac{6}{x-5} - \frac{2}{2x-3}$ h) $\frac{2}{r-s} + \frac{3}{r+s}$

i) $5 - p + \frac{3}{p-1}$ j) $2v + 1 - \frac{v^2+6}{2v+1}$

3 Simplify.

a) $\frac{y^2-2y-3}{y^2+3y+2}$

b) $\frac{e^2-2e-3}{e^2+5e+4}$

4 a) Simplify $\frac{x^2-2x-3}{x^2+3x+2}$.

b) Find the value of x, if $\frac{x^2-2x-3}{x^2+3x+2} = -1$.

Multiply algebraic fractions

The steps we use to multiply fractions are similar to those we use to simplify fractions. Work through the examples to refresh your memory.

Worked example 11

Simplify.

1 $\frac{a}{3} \times \frac{2a}{5}$ 2 $\frac{3y}{4} \times \frac{2y}{15}$ 3 $\frac{4x^2y}{12ab} \times \frac{6ab^3}{8xy}$

Answers

1 The numerator and the denominator do not share like terms.
So, multiply the terms in the numerator and the terms in the denominator:
$\frac{a}{3} \times \frac{2a}{5} = \frac{2a^2}{15}$

2 The numerator and the denominator have common fractions. Simplify the fractions first by dividing by common terms and then multiplying.
$\frac{3y}{4} \times \frac{2y}{15}$
$\frac{{}^1\cancel{3}y}{{}_2\cancel{4}} \times \frac{{}^1\cancel{2}y}{{}_5\cancel{15}} = \frac{y^2}{10}$

Worked example 1

3 The numerator an
algebraic terms. Yo

$\frac{4x^2y}{12ab} \times \frac{6ab^3}{8xy}$
$= \frac{4x^{\cancel{2}}\cancel{y}}{12\cancel{a}\cancel{b}} \times \frac{6\cancel{a}b^{\cancel{3}2}}{8\cancel{x}\cancel{y}}$
$= \frac{{}^1\cancel{4}x}{{}_2\cancel{12}} \times \frac{{}^1\cancel{6}b^2}{{}_2\cancel{8}}$
$= \frac{xb^2}{4}$

Activity 11

Simplify the following.

1 $\frac{x}{3} \times \frac{2x}{3}$ 2

5 $\frac{7ab}{3a^2} \times \frac{9ab^2}{42ab}$ 6

9 $\frac{9p^2q}{12mn} \times \frac{6m^2n}{15p^2q^2}$ 10

Divide algebraic

To divide with fractions,
the division sign (the di

Examples:

$4 \div \frac{1}{2} = 4 \times 2 = 8$

Think of cutting 4 apple
Think of dividing half a

$\frac{1}{2} \div 4 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Worked example 1

Simplify.

1 $\frac{1}{2}a \div \frac{1}{4}a$

4 $\left(\frac{3}{4}a + \frac{2}{3}a\right) \div \left(\frac{1}{3}a -\right.$

Answers

1 $\frac{1}{2}a \div \frac{1}{4}a = \frac{a}{2} \div \frac{a}{4}$
$= \frac{a}{2} \times \frac{4}{a}$
$= 2$

d) $\frac{5}{b} - \frac{4}{b}$

h) $\frac{2e-3}{5} + 2e$

terms.

d) $\frac{4}{y-5} - \frac{2}{3y+7}$

h) $\frac{2}{r-s} + \frac{3}{r+s}$

e we use to simplify
emory.

3 $\frac{4x^2y}{12ab} \times \frac{6ab^3}{8xy}$

like terms.
rms in the denominator.

fractions. Simplify the
n multiplying.

Worked example 11 (continued)

3 The numerator and the denominator contain common numerical and algebraic terms. You can divide with like terms.

$\frac{4x^2y}{12ab} \times \frac{6ab^3}{8xy}$ Divide with common factors.

$= \frac{4x^2\not{y}}{12\not{a}\not{b}} \times \frac{6\not{a}b^{\not{3}2}}{8\not{x}\not{y}}$

$= \frac{{}^{1}\not{4}x}{{}_{2}\not{12}} \times \frac{{}^{1}\not{6}b^2}{{}_{2}\not{8}}$ Then divide with numerical factors.

$= \frac{xb^2}{4}$

Activity 11

Simplify the following.

1 $\frac{x}{3} \times \frac{2x}{3}$ 2 $\frac{3y}{4} \times \frac{y}{5}$ 3 $\frac{6x}{5} \times \frac{10x^2}{12}$ 4 $\frac{5a}{a^2} \times \frac{3b}{15b^2}$

5 $\frac{7ab}{3a^2} \times \frac{9ab^2}{42ab}$ 6 $\frac{15xy}{3x^2b} \times \frac{6ab^2}{5ab}$ 7 $\frac{8y}{7} \times \frac{x}{16}$ 8 $\frac{5x^3}{5x^2} \times \frac{10x^2}{12x}$

9 $\frac{9p^2q}{12mn} \times \frac{6m^2n}{15p^2q^2}$ 10 $\frac{14p^2q}{15mn} \times \frac{6m^2n}{7p^2q^2}$

Divide algebraic fractions

To divide with fractions, **invert** the fraction after the division sign (the **divisor**) and multiply.

New words

invert: turn upside down
divisor: term that follows the division sign

Examples:

$4 \div \frac{1}{2} = 4 \times 2 = 8$

Think of cutting 4 apples cut into halves. You will have 8 halves.
Think of dividing half an apple into four pieces. Each piece will be $\frac{1}{8}$ of an apple.

$\frac{1}{2} \div 4 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Worked example 12

Simplify.

1 $\frac{1}{2}a \div \frac{1}{4}a$ 2 $\frac{15x}{3y^2} \div \frac{3x}{6y^2}$ 3 $\frac{7ab}{3a^2} \div \frac{35ab^2}{27a^3}$

4 $\left(\frac{3}{4}a + \frac{2}{3}a\right) \div \left(\frac{1}{3}a - \frac{1}{4}a\right)$ 5 $\left(\frac{2ab}{3} + \frac{3ab}{5}\right) \div \frac{38ab}{5}$

Answers

1 $\frac{1}{2}a \div \frac{1}{4}a = \frac{a}{2} \div \frac{a}{4}$

$= \frac{a}{2} \times \frac{4}{a}$ Invert the divisor. Replace ÷ with ×.

$= 2$

Worked example 12 (continued)

2 $\frac{15x}{3y^2} \div \frac{3x}{6y^2} = \frac{15x}{3y^2} \times \frac{6y^2}{3x}$ Invert the divisor and multiply.

$= \frac{{}^{5}\cancel{15}\cancel{x}}{\cancel{3}\cancel{y^2}} \times \frac{{}^{2}\cancel{6}\cancel{y^2}}{{}_{1}\cancel{3}\cancel{x}}$ Simplify by dividing by numerical and algebraic factors.

$= 10$

3 $\frac{7ab}{3a^2} \div \frac{35ab^2}{27a^3} = \frac{7ab}{3a^2} \times \frac{27a^3}{35ab^2}$

$= \frac{7a\cancel{b}}{3\cancel{a^2}} \times \frac{27\cancel{a^3}}{35\cancel{a}b^{\cancel{2}}}$ Divide with algebraic factors.

$= \frac{{}^{1}\cancel{7}a}{{}_{1}\cancel{3}} \times \frac{\cancel{27}^{9}}{{}_{5}\cancel{35}b}$ Divide with numerical factors.

$= \frac{9a}{5b}$

4 $\left(\frac{3}{4}a + \frac{2}{3}a\right) \div \left(\frac{1}{3}a - \frac{1}{4}a\right) = \left(\frac{9a + 8a}{12}\right) \div \left(\frac{4a - 3a}{12}\right)$ Simplify the brackets.

$= \frac{17a}{12} \div \frac{a}{12}$

$= \frac{17a}{12} \times \frac{12}{a}$

$= 17$

5 $\left(\frac{2ab}{3} + \frac{3ab}{5}\right) \div \frac{38ab}{5} = \frac{10ab + 9ab}{15} \div \frac{38ab}{5}$ Simplify the brackets.

$= \frac{19ab}{15} \div \frac{38ab}{5}$

$= \frac{{}^{1}\cancel{19}\cancel{ab}}{{}_{3}\cancel{15}} \times \frac{{}^{1}\cancel{5}}{{}_{2}\cancel{38}\cancel{ab}}$

$= \frac{1}{6}$

Activity 12

Simplify.

1 $\frac{1}{3}a \div \frac{1}{6}a$

2 $\frac{1}{5}x^2 \div \frac{1}{2}x$

3 $\frac{m^2}{f} \div \frac{m}{f^2}$

4 $\frac{8}{f} \div \frac{4}{f^2}$

5 $\frac{9}{2a} \div \frac{3}{4a^2}$

6 $\frac{12p}{5q} \div \frac{4p}{15q^2}$

7 $\frac{5x^2y}{3a^2} \div \frac{12xy}{27a^3}$

8 $\frac{15xy}{8xy} \div \frac{45ab^2}{32x^2y^2}$

9 $\left(\frac{2a}{5} + \frac{2a}{3}\right) \div \frac{32a}{30}$

10 $\left(\frac{4}{5}m^2 + \frac{2}{3}m^2\right) \div \left(\frac{4}{5}m - \frac{2}{3}m\right)$

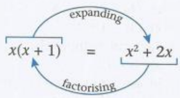
Summary

- We can only simplif
  Example: $6p - 12p +$
- We cannot simplify a
  Example: $6m + 4n$
- Expand an expressio
  brackets by every ter
  Example: $a(5 - 2a) +$
- Simplify expressions
  Example: $10a - 2a^2 +$
- Factorising is the op
  - When expanding
    the brackets
  - When factorising
    factors. Sometime
    the factors of an e
- Factorise algebraic e
  - finding common f
    Example: $35t + 30$
  - grouping like term
  - finding factors of
    Example: $3a^2 - 10$
  - finding the differe
    Examples: $4^2 - 3^2 =$
    $a^2 - 2^5 =$
- Simplify algebraic fra
  - You can use additi
    Example: $\frac{1}{2b} + \frac{1}{b}$
  - When multiplying
    Example: $\frac{2a}{4b} \times \frac{2}{a} =$
  - When dividing, in
    Example: $\frac{4a}{25} \div \frac{a^2}{5}$

Summary, revision and assessment

Summary

- We can only simplify an expression that contains like terms.
  Example: $6p - 12p + 8p = 2p$ All the terms contain p and so they are like terms.
- We cannot simplify an expression that contains unlike terms only.
  Example: $6m + 4n$ $6m$ and $4n$ are unlike terms.
- Expand an expression by removing brackets. Multiply every term outside the brackets by every term inside the brackets.
  Example: $a(5 - 2a) + a(5a - 4) = 10a - 2a^2 + 5a^2 - 4a$
- Simplify expressions by grouping like terms.
  Example: $10a - 2a^2 + 5a^2 - 4a = (10a - 4a) + (-2a^2 + 5a^2)$
  $= 6a + 3a^2$
- Factorising is the opposite of expanding:
  - When expanding an expression, you remove the brackets
  - When factorising an expression, you find the factors. Sometimes, you use brackets to write the factors of an expression.



- Factorise algebraic expressions by:
  - finding common factors
    Example: $35t + 30t^2 = 5t(7 - 6t)$
  - grouping like terms (see example above)
  - finding factors of quadratic expressions
    Example: $3a^2 - 10a + 8 = (3a - 4)(a - 2)$
    $3a = 4$ or $a = 2$
    $a = \frac{4}{3}$
  - finding the difference of two squares
    Examples: $4^2 - 3^2 = (4 + 3)(4 - 3) = 7 \times 1 = 7$
    $a^2 - 2^5 = (a + 5)(a - 5)$; $a = -5$ or $a = 5$
- Simplify algebraic fractions:
  - You can use addition and subtraction when the denominators are the same.
    Example: $\frac{1}{2b} + \frac{1}{b} - \frac{4}{3b} = \frac{3 + 6 - 8}{6b} = \frac{1}{6b}$
  - When multiplying, you can divide by common factors.
    Example: $\frac{2a}{4b} \times \frac{2}{a} = \frac{1}{a}$
  - When dividing, invert the fraction after the division sign and then multiply.
    Example: $\frac{4a}{25} \div \frac{a^2}{5} = \frac{4\cancel{a}}{\cancel{25}_5} \times \frac{\cancel{5}^1}{a^{\cancel{2}}} = \frac{4}{5a}$

Revision and assessment continued

- To simplify fractions, start by finding the LCM of the denominators. Then convert all the fractions so that they have the same denominator.

Revision exercises

1 Simplify the following by grouping like terms.
   a) $11p + 3q + p$
   b) $5a + 2b + a - 2b$

2 Expand and simplify each expression.
   a) $p(4 + p)$
   b) $ab(2a - b)$
   c) $(x - y)^2$
   d) $3(-n) - 4(2 - n)$
   e) $(3a + 2b)(a + 5)$
   f) $(3y - 1)^2$

3 Factorise completely.
   a) $4y - 12$
   b) $a^2 - ab$
   c) $ab - 2b + 3ab$
   d) $a^2 - 16^2$

4 Factorise each expression.
   a) $p^2 + 7p + 10$
   b) $b^2 - 5b + 4$
   c) $12 - y - y^2$
   d) $3x^2 - 8x - 16$
   e) $15 - 4x - 3x^2$
   f) $4x^2 + 14x + 10$

5 Express each expression as a single fraction in its simplest terms.
   a) $\frac{1}{2b} + \frac{1}{2b}$
   b) $\frac{2a}{3} - \frac{4a}{6}$
   c) $\frac{4}{m - 3} + \frac{2}{m - 2}$
   d) $\frac{y^2 + y - 12}{y^2 - 2y - 3}$

Assessment e

1 Simplify the fol
   a) $7p + 2q + 3p$
   c) $7a + 4b + 5a$

2 Expand and sim
   a) $p(2 - p)$
   c) $(2x - 3y)^2$
   e) $-3(-n) - 3(1$
   g) $(5f + 3)(2f -$

3 Factorise compl
   a) $5y - 10$
   c) $ar - 2r + 3ar$
   e) $121 - x^2$

4 If $A = b^2 - c^2$, wr
   Considering tha
   $A = 2\ 304$.

5 Factorise the fol
   a) $ry - 6 - 2y +$

6 Factorise each ex
   a) $m^2 + 7m + 10$
   c) $t^2 - t - 20$
   e) $2x^2 + x - 3$
   g) $15 - 4x - 3x^2$

7 a) $\left(\frac{2}{3}f - \frac{1}{6}f\right) \times$

8 Express each exp
   a) $\frac{1}{n} + \frac{1}{n}$
   c) $1 + \frac{2}{x} + \frac{3}{2x}$
   e) $\frac{2}{x - 3} + \frac{1}{x - 2}$
   g) $\frac{x^2 + x - 12}{x^2 - 2x - 3}$

Assessment exercises

1 Simplify the following by grouping like terms.

a) $7p + 2q + 3p$ b) $7mn + 2xy + 3xy$

c) $7a + 4b + 5a - 3b$ d) $2m - 3x - 4x - m$

2 Expand and simplify each expression.

a) $p(2 - p)$ b) $xy(x - y)$

c) $(2x - 3y)^2$ d) $\frac{2}{f}(f^2 - 3f)$

e) $-3(-n) - 3(1 - n)$ f) $(2a + 3b)(a + 7)$

g) $(5f + 3)(2f - 6) - (7f + 2)$ h) $(y - 3)^2 - (y + 3)^2$

3 Factorise completely.

a) $5y - 10$ b) $h^2 - gh$

c) $ar - 2r + 3ar$ d) $y^2 - 4^2$

e) $121 - x^2$ f) $2x^2 - 8$

4 If $A = b^2 - c^2$, write A as a product of two factors.
Considering that $2\,304 = 64 \times 36$, find the positive values of b and c for which $A = 2304$.

5 Factorise the following.

a) $ry - 6 - 2y + 3r$ b) $xy - y^2 - ry + rx$

6 Factorise each expression.

a) $m^2 + 7m + 10$ b) $h^2 - 5h + 4$

c) $t^2 - t - 20$ d) $12 - y - y^2$

e) $2x^2 + x - 3$ f) $3x^2 - 8x - 16$

g) $15 - 4x - 3x^2$ h) $7x^2 + 13x + 6$

7 a) $\left(\frac{2}{3}f - \frac{1}{6}f\right) \times \left(\frac{1}{2}a + \frac{1}{4}a\right)$ b) $\dfrac{\frac{1}{2}p + \frac{1}{3}p}{\frac{1}{6}p^2}$

8 Express each expression as a single fraction in its simplest terms.

a) $\frac{1}{n} + \frac{1}{n}$ b) $\frac{2x}{3} - \frac{x}{6}$

c) $1 + \frac{2}{x} + \frac{3}{2x}$ d) $x - \frac{x - 2}{7} - \frac{x - 1}{3}$

e) $\frac{2}{x - 3} + \frac{1}{x - 2}$ f) $\frac{3}{x + 4} - \frac{4}{x + 3}$

g) $\frac{x^2 + x - 12}{x^2 - 2x - 3}$ h) $\frac{x^2 + x - 12}{x^2 - 4x + 3}$

TOPIC 4

Matrices

Sub-topics	Specific Outcomes
Transpose of a matrix	• Find a transpose of a matrix
Multiplication of matrices	• Multiply matrices (up to 3 × 3 matrices) • Calculate the determinant of a 2 × 2 matrix
The inverse of a matrix	• Find the inverse of a 2 × 2 matrix • Solve systems of linear equations in two variables • Use matrices to solve real-life problems

Starter activity

Last year, you studied a few properties of matrices. Use this activity to refresh your knowledge of matrices. Work in pairs.

1 A CD shop sold 75 rap CDs and 89 classical CDs in October; 98 rap CDS and 81 classical CDs in November and 134 rap and 102 classical CDs in December. Represent the information in matrix form and label the rows and columns appropriately.

2 Give the order of each matrix.

a) $\begin{bmatrix} 2 & 1 \\ 4 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 5 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 5 & 9 \\ 4 & 10 & 8 \\ 11 & 7 & 13 \end{bmatrix}$

3 Multiply matrix A = $\begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$ by matrix B = [4 3 1].

SUB-TOPIC 1 Tra

Revise matric

Before learning what is
matrix, revise matrices
explanation below.

Matrices are used to
Look at the standings o

	Played	W
Team A	5	
Team B	5	
Team C	5	
Team D	5	

We can present this inf
below:

$\begin{bmatrix} 5 & 4 & 0 & 1 & 12 \\ 5 & 3 & 1 & 1 & 10 \\ 5 & 3 & 0 & 2 & 9 \\ 5 & 2 & 2 & 1 & 8 \end{bmatrix}$

- A **matrix** is an array
  and columns.
  For example, the ma
  The matrix $\begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix}$ has
  Each member of an a
- **Order of a matrix**: T
  gives the order of a m
  Examples:
  $\begin{bmatrix} 7 \\ 2 \end{bmatrix}$ has two rows an
  $\begin{bmatrix} 4 & 5 \\ 1 & 7 \end{bmatrix}$ has two rows
  $\begin{bmatrix} 4 & 5 & -2 \\ 1 & 7 & 3 \end{bmatrix}$ has two r
- A **row matrix** has o
  Example: [5 4]
- A **column matrix** ha
  Example: $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

SUB-TOPIC 1 Transpose of a matrix

Revise matrices

Before learning what is meant by the transpose of a matrix, revise matrices by working through the explanation below.

New word

matrices: plural of matrix

Matrices are used to store or display information. Look at the standings of football teams in a league.

	Played	Won	Drawn	Lost	Points
Team A	5	4	0	1	12
Team B	5	3	1	1	10
Team C	5	3	0	2	9
Team D	5	2	2	1	8

We can present this information in a matrix as shown below:

$$\begin{bmatrix} 5 & 4 & 0 & 1 & 12 \\ 5 & 3 & 1 & 1 & 10 \\ 5 & 3 & 0 & 2 & 9 \\ 5 & 2 & 2 & 1 & 8 \end{bmatrix}$$

- A **matrix** is an array of numbers that are arranged in rows and columns.
  For example, the matrix $\begin{bmatrix} 2 & 4 & 5 \\ 7 & 5 & 8 \end{bmatrix}$ has two rows and three columns.
  The matrix $\begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix}$ has three rows and one column.
  Each member of an array is called an element or an entry.
- **Order of a matrix**: The number of rows followed by the number of columns gives the order of a matrix.
  Examples:
  $\begin{bmatrix} 7 \\ 2 \end{bmatrix}$ has two rows and one column. It is a 2 × 1 matrix. (Read × as by.)
  $\begin{bmatrix} 4 & 5 \\ 1 & 7 \end{bmatrix}$ has two rows and two columns. It is a 2 × 2 matrix.
  $\begin{bmatrix} 4 & 5 & -2 \\ 1 & 7 & 3 \end{bmatrix}$ has two rows and three columns. It is a 2 × 3 matrix.
- A **row matrix** has only one row.
  Example: $[5 \quad 4]$
- A **column matrix** has only one column.
  Example: $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

- A **square matrix** has the same number of rows as columns.

  Examples:

  $\begin{bmatrix} 4 & 5 \\ 1 & 7 \end{bmatrix}$ $\begin{bmatrix} 2 & 5 & 7 \\ 3 & 1 & 6 \\ 6 & 1 & 4 \end{bmatrix}$

  All square matrices have a leading diagonal and a trailing diagonal:
  The leading diagonal runs from top left to bottom right (4, 7, and 2, 1 and 4 in the examples).
  The trailing diagonal runs from bottom left to top right (1, 5 and 6, 1 and 7 in the examples.

- A **scalar** is a quantity that has size only. When you multiply a matrix by a scalar, you multiply every element in the matrix by the scalar.

> **New word**
>
> **scalar:** a quantity that has magnitude (size) only

Example:

If $A = \begin{bmatrix} 5 & 4 \\ 4 & 6 \end{bmatrix}$, then $2A = 2 \times \begin{bmatrix} 5 & 4 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 2 \times 5 & 2 \times 4 \\ 2 \times 4 & 2 \times 6 \end{bmatrix} = \begin{bmatrix} 10 & 8 \\ 8 & 12 \end{bmatrix}$

- Matrices are equal if their corresponding elements are equal.

  Example: $A = \begin{bmatrix} 5 & 4 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 4 \\ 4 & 6 \end{bmatrix}$, therefore A and B are equal matrices.

Worked example 1

$P = \begin{bmatrix} 3x & 6 \\ 8 & -4y \end{bmatrix}$ and $Q = \begin{bmatrix} 12 & 6 \\ 8 & 10 \end{bmatrix}$. Find the values of x and y for which P = Q.

Answer

P = Q, which means $\begin{bmatrix} 3x & 6 \\ 8 & -4y \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ 8 & 10 \end{bmatrix}$ and corresponding elements are equal, therefore $3x = 12$ and $-4y = 10$. Therefore $x = 4$ and $y = -2\frac{1}{2}$.

Activity 1

1 Write down the order of each matrix.

a) $A = [3 \ -2]$

b) $B = \begin{bmatrix} 6 & 8 \\ 7 & 7 \end{bmatrix}$

c) $C = \begin{bmatrix} 7 & 2 \\ 3 & 5 \\ 4 & 3 \end{bmatrix}$

d) $D = \begin{bmatrix} 6 & 4 & 3 \\ 2 & 9 & 1 \end{bmatrix}$

e) $E = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$

f) $F = \begin{bmatrix} 1 & 4 & 0 \\ 8 & 1 & 0 \\ 0 & 6 & 1 \end{bmatrix}$

2 Which matrices in question 1 are column matrices, row matrices and square matrices?

3 Give the trailing diagonal of matrix F.

4 Give the leading diagonal of (a) matrix B and (b) matrix F.

5 If matrix B above equals $Q = \begin{bmatrix} 3x & 8 \\ 14y & 7 \end{bmatrix}$, find the values of x and y.

Find a transpo

We use the symbol A^T to
of a matrix A. To find th
any matrix A, interchan
and columns.

Example: Transpose mat

The answer is $\begin{bmatrix} 6 & 2 \\ 4 & 9 \\ 3 & 1 \end{bmatrix}$.

Activity 2

1 Give the order of ea

a) $A = [3 \ -2]$

b) $B = \begin{bmatrix} 6 & 8 \\ 7 & 7 \end{bmatrix}$

c) $C = \begin{bmatrix} 7 & 2 \\ 3 & 5 \\ 4 & 3 \end{bmatrix}$

d) $D = \begin{bmatrix} 6 & 4 & 3 \\ 2 & 9 & 1 \end{bmatrix}$

e) $E = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$

f) $F = \begin{bmatrix} 1 & 4 & 0 \\ 8 & 1 & 0 \\ 0 & 6 & 1 \end{bmatrix}$

2 Find the transpose o

3 Write down the orde
question 1.

4 Find the transpose o

a) $\begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 6 & 2 \\ 4 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

Find a transpose of a matrix

We use the symbol A^T to show the transpose of a matrix A. To find the transpose (A^T) of any matrix A, interchange (swap) the rows and columns.

Example: Transpose matrix $\begin{bmatrix} 6 & 4 & 3 \\ 2 & 9 & 1 \end{bmatrix}$.

The answer is $\begin{bmatrix} 6 & 2 \\ 4 & 9 \\ 3 & 1 \end{bmatrix}$.

Note

- The first row of A becomes the first column of A^T.
- The second row of A becomes the second column of A^T.
- The order of A is 2×3 and the order of A^T is 3×2.

Activity 2

1 Give the order of each matrix.

a) $A = \begin{bmatrix} 3 & -2 \end{bmatrix}$

b) $B = \begin{bmatrix} 6 & 8 \\ 7 & 7 \end{bmatrix}$

c) $C = \begin{bmatrix} 7 & 2 \\ 3 & 5 \\ 4 & 3 \end{bmatrix}$

d) $D = \begin{bmatrix} 6 & 4 & 3 \\ 2 & 9 & 1 \end{bmatrix}$

e) $E = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$

f) $F = \begin{bmatrix} 1 & 4 & 0 \\ 8 & 1 & 0 \\ 0 & 6 & 1 \end{bmatrix}$

2 Find the transpose of each matrix in question 1.

3 Write down the order of A^T, B^T, C^T, D^T, E^T and F^T of the matrices in question 1.

4 Find the transpose of each matrix.

a) $\begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 6 & 2 \\ 4 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

SUB-TOPIC 2 Multiplication of matrices

You can multiply one matrix by another matrix if the number of columns in the first matrix is the same as the number of rows in the second matrix. You can then multiply the elements of each row in the first matrix with each element in each column in the second matrix.

Multiply matrices of order up to 2 by 2

The example shows how to multiply matrices of order 2 by 2.

Worked example 2

If $A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ and $B = \begin{bmatrix} w & y \\ x & z \end{bmatrix}$, find the product AB.

Answer

$$AB = \begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} w & y \\ x & z \end{bmatrix} = \begin{bmatrix} a \times w + c \times x & a \times y + c \times z \\ b \times w + d \times x & b \times y + d \times z \end{bmatrix} = \begin{bmatrix} aw + cx & ay + cz \\ bw + dx & by + dz \end{bmatrix}$$

- The order of the matrix that is obtained when two matrices are multiplied is determined as follows:

2 by 2
× 2 by 2 gives a 2 × 2 matrix; for example, $\begin{bmatrix} 4 & 5 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 37 & 60 \\ 55 & 89 \end{bmatrix}$

3 by 2
× 2 by 2 gives a 3 × 2 matrix; for example, $\begin{bmatrix} 4 & 2 \\ 5 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 6 & 9 \end{bmatrix} = \begin{bmatrix} 28 & 39 \\ 38 & 43 \\ 16 & 23 \end{bmatrix}$

2 by 1
× 1 by 2 gives a 2 × 2 matrix; for example, $\begin{bmatrix} 5 \\ 2 \end{bmatrix}\begin{bmatrix} 9 & 5 \end{bmatrix} = \begin{bmatrix} 45 & 25 \\ 18 & 10 \end{bmatrix}$

1 by 3
× 3 by 1 gives a 1 × 1 matrix; for example, $\begin{bmatrix} 2 & 3 & 5 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \\ 6 \end{bmatrix} = 53$

Calculation: $(2 \times 7) + (3 \times 3) + (5 \times 6) = 53$

Activity 3

Work with a partner.

1 If $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, find the following.

a) AB
b) BA
c) the order of AB
d) the order of BA

Activity 3 (continued

2 If $M = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and N =

a) MN
b) NM
c) the order of MN
d) the order of NM

3 What do you notice a
question 2? Discuss w

4 Find the product: $\begin{bmatrix} 7 \\ 3 \\ 3 \end{bmatrix}$

Note

AB ≠ BA in question 1 in Ac
usually the case because m
multiplication is not comm

The identity matri

The identity matrix is also
diagonal in an **identity** m
Examples:

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

If you multiply a matrix b
is the same as when you m

Activity 4

1 Multiply each matrix

a) $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

2 Multiply the two mat

a) $\begin{bmatrix} 3 & -2 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The zero (0) matri

All the elements in a zero

Examples: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

When you multiply a mat

Activity 3 (continued)

2 If $M = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $N = [3 \quad -2]$, find the following.

a) MN

b) NM

c) the order of MN

d) the order of NM

3 What do you notice about AB and BA in question 1, and NM and MN in question 2? Discuss with a partner.

4 Find the product: $\begin{bmatrix} 7 & 2 \\ 3 & 5 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$.

Note

AB ≠ BA in question 1 in Activity 3; this is usually the case because matrix multiplication is not **commutative**.

New word

commutative: changing the order of the numbers in an operation does not change the answer (for example, $2 \times 3 = 3 \times 2$)

The identity matrix

The identity matrix is also called the unit matrix. The elements in the leading diagonal in an **identity matrix** are all 1. All other elements are 0.

Examples:

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

If you multiply a matrix by the identity matrix, the matrix does not change. This is the same as when you multiply a number by 1.

Activity 4

1 Multiply each matrix by an identity matrix. Show all your calculations.

a) $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$ b) $B = \begin{bmatrix} 1 & 4 & 3 \\ 8 & 1 & 7 \\ 9 & 6 & 1 \end{bmatrix}$

2 Multiply the two matrices.

a) $\begin{bmatrix} 3 & -2 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 7 & 4 \end{bmatrix}$

The zero (0) matrix

All the elements in a zero matrix are equal to 0.

Examples: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

When you multiply a matrix by the zero matrix, the answer is a zero matrix.

Activity 5

Multiply each matrix by a zero matrix. Show all your calculations.

1 $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$ 2 $B = \begin{bmatrix} 1 & 4 & 3 \\ 8 & 1 & 7 \\ 9 & 6 & 1 \end{bmatrix}$

Multiply matrices of order 3 by 3

We multiply 3 by 3 matrices in the same way as 2 by 2 matrices.

Worked example 3

If $M = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 9 \\ 7 & 5 & 8 \end{bmatrix}$ and $N = \begin{bmatrix} 1 & 3 & 5 \\ 5 & 2 & 1 \\ 6 & 5 & 2 \end{bmatrix}$ find the following matrix products.

1 MN 2 NM

Answers

1 $$MN = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 9 \\ 7 & 5 & 8 \end{bmatrix}\begin{bmatrix} 1 & 3 & 5 \\ 5 & 2 & 1 \\ 6 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1+3\times5+5\times6 & 2\times3+3\times2+5\times5 & 2\times5+3\times1+5\times2 \\ 4\times1+6\times5+9\times6 & 4\times3+6\times2+9\times5 & 4\times5+6\times1+9\times2 \\ 7\times1+5\times5+8\times6 & 7\times3+5\times2+8\times5 & 7\times5+5\times1+8\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 47 & 37 & 23 \\ 88 & 69 & 44 \\ 80 & 71 & 56 \end{bmatrix}$$

2 $$NM = \begin{bmatrix} 1 & 3 & 5 \\ 5 & 2 & 1 \\ 6 & 5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 9 \\ 7 & 5 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2+3\times4+5\times7 & 1\times3+3\times6+5\times5 & 1\times5+3\times9+5\times8 \\ 5\times2+2\times4+1\times7 & 5\times3+2\times6+1\times5 & 5\times5+2\times9+1\times8 \\ 6\times2+5\times4+2\times7 & 6\times3+5\times6+2\times5 & 6\times5+5\times9+2\times8 \end{bmatrix}$$

$$= \begin{bmatrix} 49 & 46 & 72 \\ 25 & 32 & 51 \\ 46 & 58 & 91 \end{bmatrix}$$

Note

MN ≠ NM; this shows that matrix multiplication is not commutative.

Activity 6

Multiply each pair of matrices.

1 $\begin{bmatrix} 4 & 5 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ 2 $\begin{bmatrix} 4 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ 3 $\begin{bmatrix} 4 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ -6 \end{bmatrix}$

4 $\begin{bmatrix} 5 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ -4 \end{bmatrix}$ 5 $\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} -1 & -4 \end{bmatrix}$ 6 $\begin{bmatrix} 2 \\ 20 \end{bmatrix}\begin{bmatrix} 4 & 6 \end{bmatrix}$

7 $\begin{bmatrix} 12 \\ 10 \end{bmatrix}\begin{bmatrix} -2 & -4 \end{bmatrix}$ 8 $\begin{bmatrix} 2 \\ -6 \end{bmatrix}\begin{bmatrix} -3 & -3 \end{bmatrix}$ 9 $\begin{bmatrix} 2 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 1 & 1 \end{bmatrix}$

Activity 6 (continu

10 $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$

13 $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$

16 $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

19 $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 12 & 1 \\ 9 & 10 & 3 \end{bmatrix}$

Calculate the

The determinant of a 2
the product of the elem
elements of the leadin
If $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, the deter
$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

Worked example 4

Find the determinant

1 $\begin{bmatrix} 4 & 2 \\ 6 & 4 \end{bmatrix}$

2 $\begin{bmatrix} 2 & -2 \\ 4 & 4 \end{bmatrix}$

Answers

1 $\begin{vmatrix} 4 & 2 \\ 6 & 4 \end{vmatrix}$ det. = 4 ×

2 $\begin{vmatrix} 2 & -2 \\ 4 & 4 \end{vmatrix}$ det. = 2 ×

Activity 7

Find the determinant o

1 $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$

4 $\begin{bmatrix} 11 & 4 \\ 6 & 2 \end{bmatrix}$

7 $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

10 $\begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$

Activity 6 (continued)

10 $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$ 11 $\begin{bmatrix} 4 & 3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}$ 12 $\begin{bmatrix} 2 & 7 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

13 $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$ 14 $\begin{bmatrix} -3 & 2 \\ -2 & 15 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ -2 & 6 \end{bmatrix}$ 15 $\begin{bmatrix} -1 & -4 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

16 $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ 17 $\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}$ 18 $\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}\begin{bmatrix} 2 & 1 & 5 \end{bmatrix}$

19 $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 12 & 1 \\ 9 & 10 & 3 \end{bmatrix}$ 20 $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 6 & 9 \\ 5 & 0 & 8 \end{bmatrix}\begin{bmatrix} 2 & 3 & 8 \\ 1 & 1 & 6 \\ 4 & 3 & 3 \end{bmatrix}$ 21 $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 8 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 & 0 \\ 3 & -4 & 6 \\ 2 & 4 & -3 \end{bmatrix}$

Calculate the determinant of a 2 by 2 matrix

The determinant of a 2 by 2 matrix is a number. Find this number by subtracting the product of the elements of the trailing diagonal from the product of the elements of the leading diagonal.

If $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, the determinant of A, which we write as |A| or det. A is given by:

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

Worked example 4

Find the determinants of each matrix.

1 $\begin{bmatrix} 4 & 2 \\ 6 & 4 \end{bmatrix}$

2 $\begin{bmatrix} 2 & -2 \\ 4 & 4 \end{bmatrix}$

Answers

1 $\begin{vmatrix} 4 & 2 \\ 6 & 4 \end{vmatrix}$ det. = 4 × 4 − 2 × 6 = 2

2 $\begin{vmatrix} 2 & -2 \\ 4 & 4 \end{vmatrix}$ det. = 2 × 4 − (4 × −2) = 16

Activity 7

Find the determinant of each matrix.

1 $\begin{bmatrix} 9 & 5 \\ 3 & 2 \end{bmatrix}$ 2 $\begin{bmatrix} 10 & 15 \\ 2 & 3 \end{bmatrix}$ 3 $\begin{bmatrix} 7 & 3 \\ 9 & 5 \end{bmatrix}$

4 [illegible] $\begin{bmatrix} \cdot & 4 \\ \cdot & 2 \end{bmatrix}$ 5 $\begin{bmatrix} 14 & 7 \\ 4 & 2 \end{bmatrix}$ 6 $\begin{bmatrix} 1 & 15 \\ 2 & 33 \end{bmatrix}$

7 [illegible] $\begin{bmatrix} \cdot & \cdot \\ \cdot & 3 \end{bmatrix}$ 8 $\begin{bmatrix} -4 & -2 \\ -5 & 3 \end{bmatrix}$ 9 $\begin{bmatrix} -3 & -1 \\ -5 & -3 \end{bmatrix}$

10 [illegible] $\begin{bmatrix} \cdot & 2 \\ \cdot & 3 \end{bmatrix}$ 11 $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$ 12 $\begin{bmatrix} 6 & 2 \\ 4 & 0 \end{bmatrix}$

Use a determinant to find an unknown value

We can use the determinant of a matrix to find an unknown value in the matrix.

Worked example 5

The value of the determinant of the matrix $\begin{bmatrix} k-1 & 3 \\ -2 & k \end{bmatrix}$ is 12. Find the possible values of k.

Answer

det. $\begin{bmatrix} k-1 & 3 \\ -2 & k \end{bmatrix} = 12$

This means:

$(k-1)k - 3 \times (-2) = 12$

$\therefore k^2 - k + 6 = 12$

$\therefore k^2 - k - 6 = 0$

$(k+2)(k-3) = 0$

$\therefore k = -2$ or $k = 3$

Therefore, the possible values of k are -2 or 3.

Activity 8

You may work in pairs.

1 The determinant of matrix $\begin{bmatrix} k & 3 \\ k & -2 \end{bmatrix}$ is 15. Find the possible values of k.

2 Find the value of the determinant $\begin{bmatrix} 5 & 3 \\ k-3 & k \end{bmatrix}$ if k takes the following values.

a) $k = 7$
b) $k = -2$
c) $k = 4.5$
d) $k = 0$

3 Find the value(s) of k for which matrix $\begin{bmatrix} k-1 & 3 \\ 2 & k \end{bmatrix}$ has the given value.

a) 0
b) 6
c) -10

The adjoint of a 2 by 2 matrix

The adjoint of matrix A (indicated by adj. A) is the matrix that is formed by:

- interchanging (swapping) the elements of the leading diagonal
- changing the signs of the elements of the trailing diagonal.

If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then adj. A = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Worked example 6

If $A = \begin{bmatrix} 3 & -2 \\ 4 & 7 \end{bmatrix}$, give adj. A.

Answer

adj. $A = \begin{bmatrix} 7 & 2 \\ -4 & 3 \end{bmatrix}$

Activity 9

1 Write down the adjoint of each matrix.

a) $\begin{bmatrix} 9 & 7 \\ -5 & 5 \end{bmatrix}$ b) $\begin{bmatrix} -3 & -2 \\ -5 & 3 \end{bmatrix}$

c) $\begin{bmatrix} -4 & -2 \\ -5 & 3 \end{bmatrix}$ d) $\begin{bmatrix} -3 & -1 \\ -5 & -9 \end{bmatrix}$

e) $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$ f) $\begin{bmatrix} 6 & 2 \\ 4 & 0 \end{bmatrix}$

2 Matrices A, B and C are given below.

$A = \begin{bmatrix} -2 & 3 \\ 5 & -1 \end{bmatrix}$

$B = \begin{bmatrix} 8 & -3 \\ 2 & 5 \end{bmatrix}$

$C = \begin{bmatrix} 12 & 1 \\ 2 & 4 \end{bmatrix}$

Find the following.

a) adj. A b) adj. B c) adj. C

3 Refer to matrices A, B and C in question 2 and find the following.

a) $|A|$ b) $|B|$

c) $|C|$ d) $|\text{adj. } A|$

e) $|\text{adj. } B|$ f) $|\text{adj. } C|$

4 Write down the adjoint of each matrix.

a) $\begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 6 & 2 \\ 4 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 9 & 7 \\ -5 & 5 \end{bmatrix}$

e) $\begin{bmatrix} -3 & -2 \\ -5 & 3 \end{bmatrix}$ f) $\begin{bmatrix} -4 & -2 \\ -5 & 3 \end{bmatrix}$

g) $\begin{bmatrix} -3 & -1 \\ -5 & -9 \end{bmatrix}$ h) $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

5 Find the determinant of each matrix in question 4.

6 Give the transpose of each matrix in question 4.

SUB-TOPIC 3 The inverse of a matrix

If the product of two numbers x and y equals 1, in other words, $xy = 1$:

- x is the inverse of y
- y is the inverse of x.

Example: $7 \times \frac{1}{7} = 1$, so 7 is the inverse of $\frac{1}{7}$, and $\frac{1}{7}$ is the inverse of 7.

In the same way, if the product of two 2×2 matrices A and B gives the identity or unit matrix, in other words, $AB = I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$, then the one matrix is the inverse of the other matrix.

The inverse of any matrix A (written as A^{-1}) is given by the formula:

$A^{-1} = \frac{1}{\text{det. A}} \times \text{adj. A}$.

Worked example 7

1 If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ show that A is the inverse of B, and B is the inverse of A.

2 If $A = \begin{bmatrix} 4 & 3 \\ 6 & 5 \end{bmatrix}$ find the inverse of A.

Answers

1 If A is the inverse of B, and B is the inverse of A, we have to show that AB = I.

Therefore, $AB = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 \times 3 + 5 \times (-1) & 2 \times (-5) + 5 \times 2 \\ 1 \times 3 + 3 \times (-1) & 1 \times (-5) + 3 \times 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Also, $BA = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 2 + (-5) \times 1 & 3 \times 5 + (-5) \times 3 \\ -1 \times 2 + 2 \times 1 & -1 \times 5 + 2 \times 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

AB = BA = I, hence A is the inverse of B and B is the inverse of A.

2 Det. $A = 4 \times 5 - 3 \times 6 = 2$ and Adj. $A = \begin{bmatrix} 5 & -3 \\ -6 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{\text{det. A}} \times \text{adj. A}$

$= \frac{1}{2}\begin{bmatrix} 5 & -3 \\ -6 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2\frac{1}{2} & -1\frac{1}{2} \\ -3 & -2 \end{bmatrix}$

Activity 10

1 Find the inverse of

a) $\begin{bmatrix} -2 & 3 \\ 5 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 8 & -3 \\ 2 & 5 \end{bmatrix}$

c) $\begin{bmatrix} 12 & 1 \\ 2 & 4 \end{bmatrix}$

d) $\begin{bmatrix} -3 & -2 \\ 5 & 3 \end{bmatrix}$

e) $\begin{bmatrix} -3 & -5 \\ -2 & -4 \end{bmatrix}$

f) $\begin{bmatrix} -3 & -1 \\ -5 & -3 \end{bmatrix}$

2 Show that the mat

inverse of matrix A

a) $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ and

b) $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$ and

c) $A = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$ and

d) $A = \begin{bmatrix} 1 & -0.6 \\ -1 & 0.8 \end{bmatrix}$ a

3 If $A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$ and B

a) A^{-1}

b) B^{-1}

c) AA^{-1}

d) BB^{-1}

4 If $M = \begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix}$ and N

a) MM^{-1}

b) $MM^{-1}N$

c) NN^{-1}

d) $NN^{-1}M$

5 What do you notic

Activity 10

1 Find the inverse of each matrix.

a) $\begin{bmatrix} -2 & 3 \\ 5 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 8 & -3 \\ 2 & 5 \end{bmatrix}$

c) $\begin{bmatrix} 12 & 1 \\ 2 & 4 \end{bmatrix}$

d) $\begin{bmatrix} -3 & -2 \\ 5 & 3 \end{bmatrix}$

e) $\begin{bmatrix} -3 & -5 \\ -2 & -4 \end{bmatrix}$

f) $\begin{bmatrix} -3 & -1 \\ -5 & -3 \end{bmatrix}$

2 Show that the matrix A is the inverse of matrix B, and matrix B is the inverse of matrix A. In other words, show that $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if:

a) $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$

b) $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1\frac{1}{2} \\ -1 & 1 \end{bmatrix}$

c) $A = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & 5 \\ 3 & -2 \end{bmatrix}$

d) $A = \begin{bmatrix} 1 & -0.6 \\ -1 & 0.8 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 5 & 5 \end{bmatrix}$

3 If $A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 7 & 13 \end{bmatrix}$, find the following:

a) A^{-1}
b) B^{-1}
c) AA^{-1}
d) BB^{-1}

4 If $M = \begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix}$ and $N = \begin{bmatrix} 6 & 1 \\ 4 & 1 \end{bmatrix}$ find the following:

a) MM^{-1}
b) $MM^{-1}N$
c) NN^{-1}
d) $NN^{-1}M$

5 What do you notice about your results to questions 4(b) and (d)?

A singular matrix

A matrix is singular if its determinant is 0 (zero). A singular matrix does not have an inverse.

Worked example 8

1 Show that the matrix $\begin{bmatrix} 4 & 2 \\ 10 & 5 \end{bmatrix}$ is a singular matrix.

2 Find the values of p for which the matrix $A = \begin{bmatrix} p-2 & 3 \\ 5 & p \end{bmatrix}$ is singular.

Answers

1 $\begin{vmatrix} 4 & 2 \\ 10 & 5 \end{vmatrix} = 4 \times 5 - 2 \times 10 = 0$

The determinant (det.) of the matrix is 0 (zero) and so it is a singular matrix.

2 If the matrix is singular, det. A = 0

$$\text{det. } A = \begin{vmatrix} p-2 & 3 \\ 5 & p \end{vmatrix} = (p-2)p - 3 \times 5$$

$\therefore p^2 - 2p - 15 = 0$

$(p-5)(p+3) = 0$

$\therefore p = 5 \text{ or } -3$

The possible values of p are, therefore, 5 or −3.

Activity 11

1 Show that the following matrices are singular matrices.

a) $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 6 & 3 \\ 4 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$

d) $\begin{bmatrix} 3 & -4 \\ -6 & 8 \end{bmatrix}$

2 P is a 2 by 2 matrix, $\begin{bmatrix} 5 & 9 \\ 8 & 4 \end{bmatrix}$. Work in pairs to answer the questions.

a) Find the inverse matrix P^{-1}.

b) Find the matrix product PP^{-1}.

3 Calculate the value of k for which each matrix is a singular matrix.

a) $\begin{bmatrix} 5-k & 9 \\ 8 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 8 & k \\ 2 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 6 & 9 \\ 8 & k \end{bmatrix}$

d) $\begin{bmatrix} 4-k & 16 \\ -2 & k \end{bmatrix}$

Solving systems of linear equations in two variables

A useful application of matrices is solving systems of linear equations. We will limit the process to equations in two variables.

Write equations in matrix form

Look at the two equations.

$2x + y = 7$
$x + 3y = 11$

We can write the above equations in matrix form: $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$

By matrix multiplication we know that on the left side, $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x+y \\ x+3y \end{bmatrix}$

If we let $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$, we can represent the set of linear equations by AX = B.

Find the solution to a system of linear equations

The goal is to find the values of x and y that make the equations true, therefore a solution for $X = \begin{bmatrix} x \\ y \end{bmatrix}$

Worked example 9

Use matrices to solve the system of linear equations below:

$2x + y = 7$
$x + 3y = 11$

Answer

We can start with AX = B and as we know that $A^{-1}A = I$, multiply both sides by A^{-1}:

$A^{-1}AX = A^{-1}B$, so that
$X = A^{-1}B$, where A^{-1} is the inverse matrix of A.

Follow the steps to solve the equations:

Step 1 For the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, find det. $A = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5$.

Step 2 Find adj. $A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

Step 3 Find the inverse of the matrix A, using the formula:

$A^{-1} = \frac{1}{\text{det. A}} \times \text{adj. A}$

$= \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

Solving systems of linear equations in two variables

A useful application of matrices is solving systems of linear equations. We will limit the process to equations in two variables.

Write equations in matrix form

Look at the two equations.

$2x + y = 7$

$x + 3y = 11$

We can write the above equations in matrix form: $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$

By matrix multiplication we know that on the left side, $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + y \\ x + 3y \end{bmatrix}$

If we let $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$, we can represent the set of linear equations by AX = B.

Find the solution to a system of linear equations

The goal is to find the values of x and y that make the equations true, therefore a solution for $X = \begin{bmatrix} x \\ y \end{bmatrix}$

Worked example 9

Use matrices to solve the system of linear equations below:

$2x + y = 7$

$x + 3y = 11$

Answer

We can start with AX = B and as we know that $A^{-1}A = I$, multiply both sides by A^{-1}:

$A^{-1}AX = A^{-1}B$, so that

$X = A^{-1}B$, where A^{-1} is the inverse matrix of A.

Follow the steps to solve the equations:

Step 1 For the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, find det. $A = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5$.

Step 2 Find adj. $A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

Step 3 Find the inverse of the matrix A, using the formula:

$$A^{-1} = \frac{1}{\text{det. A}} \times \text{adj. A}$$

$$= \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

Worked example 9 (continued)

Step 4 Find $X = A^{-1} \cdot B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 3 \times 7 + (-1) \times 11 \\ -1 \times 7 + 2 \times 11 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$\therefore x = 2$ and $y = 3$

Note

If det. A equals 0 there is no solution to the system of equations.

Activity 12

1 Solve the following for x and y using inverse matrices.

a) $\begin{bmatrix} 6 & -2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 26 \end{bmatrix}$

2 Solve the following systems of linear equations using inverse matrices.

a) $2x + 3y = 5$
$x + 2y = 3$

b) $2x - y = 5$
$x + 2y = 4$

c) $5x + 3y = 4$
$3x + 2y = 2$

d) $x + 4y = 3$
$2x + y = -1$

e) $3x - 2y = 8$
$2x + 3y = 1$

f) $4x - 5y = -3$
$3x + 2y = -8$

Cramer's rule

Cramer's rule is a short method of finding the value of a variable in a system of linear equations. You have to work with determinants if you want to use Cramer's rule. So make sure that you know how to find the determinant of a matrix.

Using Cramer's rule to solve systems of linear equations

Worked example 10 below shows how to use Cramer's rule to solve for x and y in the system of equations from Worked example 9.

Worked example 10

Solve the system of equations:

$2x + y = 7$
$x + 3y = 11$

Note

You can only use Cramer's rule if the determinant D is not equal to 0.

Worked example 1

Answer

Step 1 Write down th

and the matri

Step 2 Find the deter

$D = 6 - 1 = 5$

Step 3 Replace the x-

Indicate it wit

Step 4 Find the deter

$D_x = 21 - 11 =$

Step 5 Calculate the

$x = \frac{D_x}{D} = \frac{10}{5} =$

Activity 13

1 Complete the calcu

equations in Work

2 Solve the following

a) $p + 5q = 12$
$3p - 2q = 4$

c) $x - y = 8$
$4x + y = 42$

e) $3m - 4n = 18$
$2m - 5n = 19$

g) $5x + 7y = 44$
$x + 3y = 12$

i) $8a + b = 20$
$11a + 4b = 17$

k) $5x + 3y = 7$
$4x + y = 7$

3 Why can you only

matrix is not equal

Apply matrice

information that c

matrix form. The matri

to translate the probl

Worked example 10 (continued)

Answer

Step 1 Write down the coefficient matrix, $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, the variable matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ and the matrix of the constants $\begin{bmatrix} 7 \\ 11 \end{bmatrix}$.

Step 2 Find the determinant (D) of the coefficient matrix, $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.
$D = 6 - 1 = 5$

Step 3 Replace the x-column in the coefficient matrix with the constants: $\begin{bmatrix} 7 & 1 \\ 11 & 3 \end{bmatrix}$. Indicate it with M_x.

Step 4 Find the determinant (D_x) of this matrix $\begin{bmatrix} 7 & 1 \\ 11 & 3 \end{bmatrix}$.
$D_x = 21 - 11 = 10$

Step 5 Calculate the value of x as follows:
$x = \frac{D_x}{D} = \frac{10}{5} = 2.$

Activity 13

1 Complete the calculation to find the value of y for the system of linear equations in Worked example 10. Use Cramer's rule.

2 Solve the following systems of linear equations using Cramer's rule.

a) $p + 5q = 12$
$3p - 2q = 4$

b) $3m + 2n = 10$
$m + 2n = 4$

c) $x - y = 8$
$4x + y = 42$

d) $3x + 2y = 12$
$2x - 3y = 7$

e) $3m - 4n = 18$
$2m - 5n = 19$

f) $3x - y = 15$
$4x + 2y = 10$

g) $5x + 7y = 44$
$x + 3y = 12$

h) $12v + 5z = -9$
$5v + z = -7$

i) $8a + b = 20$
$11a + 4b = 17$

j) $x + 4y = 42$
$2x + 5y = 57$

k) $5x + 3y = 7$
$4x + y = 7$

l) $6m + 3n = 9$
$4m + 5n = 3$

3 Why can you only use Cramer's rule if the determinant of the coefficient matrix is not equal to 0 (zero)?

Apply matrices to solve real-life problems

Any information that can be arranged in rows and columns can be represented in matrix form. The matrix can then be used to solve real-life problems. The first step is to translate the problem into a set of linear equations.

Worked example 11

Crafty Hands makes two types of printed scarf as souvenirs, A with animal prints and F with flower prints. It takes five minutes on a silk screen printer and seven minutes on a second machine to make type A scarves. It takes four minutes on a silk screen printer and five minutes on a second machine to make type F scarves. Crafty Hands can use the silk screen printer for three hours and the second machine for five hours. How many of each type of scarf should Crafty Hands make to use the machines most effectively?

Answer

We organise the information in a table.

	Type A scarf	Type F scarf	Time available (minutes)
Silk screen printer	5	5	180
Second printer	7	4	300

In order to find the number of each types of scarf to make, we let the number of type A scarf be x and the number of type F scarf be y.

Therefore, the total amount of time used on the silk screen printer is given by:

$5x + 5y = 180$

The total amount of time used on the second printer is given by:

$7x + 4y = 300$

To solve the two equations simultaneously, we can use Cramer's rule.

Step 1 Write the matrices in the form AX = B:

$$\begin{bmatrix} 5 & 5 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 180 \\ 300 \end{bmatrix}$$

Step 2 Now find det. A, $D = \begin{vmatrix} 5 & 5 \\ 7 & 4 \end{vmatrix} = 20 - 35 = -15$.

Step 3 Replace the first column in A with B and find:

$$D_x = \begin{vmatrix} 180 & 5 \\ 300 & 4 \end{vmatrix} = 720 - 1\,500 = -780$$

Step 4 Calculate: $x = \frac{D_x}{D} = \frac{-780}{-15} = 52$

Repeat steps 3 to 4 to calculate the value of y.

$$D_y = \begin{vmatrix} 5 & 180 \\ 7 & 300 \end{vmatrix}$$

$= 600 - 1\,260$

$= -660$

$y = \frac{D_y}{D}$

$= \frac{-660}{-15}$

$= 44$

Scarves for sale

Therefore, Crafty Hands should make 52 animal print scarves and 44 flower print scarves.

Activity 14

1 A shop sells plates
settings (and no se
contains eight plac
setting be Kx and th

a) Draw up two eq
b) Use matrices (an
of a serving set.

2 Chanda spends K2
buys three cool dri
cost of a cool drink
a) Use the above in
of items bought
b) Write the system
c) Use Cramer's ru
drink and a bott

3 Nosiku has K20 000
she can expect an a
the expected annua
Let the amount to i
money market fund
a) Summarise the i
b) Write an equatio
amount investe
c) Write an equatio
of the invested a
d) Use matrices (Cr
amount she sho

4 A company makes
• Materials for A c
• Materials for B c
• Materials for C c
Number of product
A: 28 units, B: 30
a) Display the cos
b) Display the nu
c) Find the matrix

Activity 14

1 A shop sells plates and cutlery at K360 for a box that contains four **place settings** (and no serving sets). They also have boxes at K880 that each contains eight place settings plus a serving set. Let the cost of one place setting be Kx and the cost of a serving set be Ky.

New word

place setting: the plates, and cup and saucer for one person for a formal meal

a) Draw up two equations – one to describe each cost.

b) Use matrices (and Cramer's rule) to calculate the cost of a place setting and of a serving set.

2 Chanda spends K29.00 on four cool drinks and two bottles of water. Malita buys three cool drinks and five bottles of water and spends K34.00. Let the cost of a cool drink be Kx and the cost of water be Ky.

a) Use the above information and write a system of equations for the number of items bought and the total cost for each person.

b) Write the system in matrix form ($AX = B$).

c) Use Cramer's rule (or use inverse matrices) to calculate the cost of a cool drink and a bottle of water.

3 Nosiku has K20 000 to invest. A friend tells her to invest in stocks for which she can expect an annual return of 12% and in a money market fund where the expected annual return is 4%. She wants an overall average return of 9%. Let the amount to invest in stocks be x and the amount to invest in the money market fund be y.

a) Summarise the information in a table.

b) Write an equation to represent the total amount invested.

c) Write an equation to represent the percentage of the invested amounts as an annual return.

d) Use matrices (Cramer's rule) to calculate what amount she should invest in each fund.

Share prices

4 A company makes products A, B and C. Unit costs for the products:

- Materials for A cost K50 per unit and K400 to produce
- Materials for B cost K80 per unit and K600 to produce
- Materials for C cost K70 per unit and K550 to produce.

Number of products produced in two months:
A: 28 units, B: 30 units and C: 34 units.

a) Display the costs of these products in a 3 by 2 matrix C.

b) Display the number of units of each product in a 1 by 3 matrix (N).

c) Find the matrix product NC and state what it represents.

TOPIC 4

Summary, revision and assessment

Summary

The transpose of a matrix

To find the transpose of a matrix:

- interchange the rows and columns of matrix A to create its transposed matrix (A^T)
- note that if the order of matrix A was $m \times n$, the order of A^T is $n \times m$.

Multiply with matrices

When multiplying with matrices:

- the number of columns of the first matrix and the number of rows of the second matrix must be the same
- matrix multiplication is not commutative
- the order of the product of an $a \times b$ matrix with a $b \times c$ matrix is $a \times c$.
- the elements of the leading diagonal of a unit or identity matrix are 1 and all the other elements of the matrix equal 0
- all the elements of a zero matrix are 0
- find the determinant by subtracting the product of the elements of the trailing diagonal from the product of the elements of the leading diagonal
- form the adjoint of a matrix by interchanging the elements of the leading diagonal and changing the signs of the trailing diagonal.

Use the inverse of a matrix

When working with the inverse of a matrix:

- write the inverse of any matrix A as A^{-1}, which is given by: $A^{-1} = \frac{1}{\text{det. A}} \times \text{adj. A}$
- only square matrices have inverses
- the determinant of a singular matrix is 0 (zero)
- square matrices that have a zero determinant do not have inverses
- solve systems of linear equations in two variables using:
  - » inverse determinants
  - » Cramer's rule (which means you can solve for one variable at a time without having to solve the whole system of equations).

Revision exe

1 Give the order of

a) $[3 \ \ 7]$

d) $\begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$

2 Transpose each m

3 Solve the followi

a) $\begin{bmatrix} 3x & 0 \\ 5 & 2y \end{bmatrix} = \begin{bmatrix} 12 \\ 5 \end{bmatrix}$

4 Find the product

a) $\begin{bmatrix} 7 & 2 \\ 3 & 5 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 6 \end{bmatrix}$

5 If $M = \begin{bmatrix} 7 & 12 \\ 10 & 4 \end{bmatrix}$ a

a) MN

6 Explain why mat the determinant

7 Find the inverse

a) $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

8 The following ma

a) $\begin{bmatrix} x & 1 \\ 4 & x \end{bmatrix}$

c) $\begin{bmatrix} x-2 & 7 \\ 5 & x \end{bmatrix}$

9 Use a matrix met

a) $p + 5q = 12$

$3p - 2q = 2$

10 A street vendor h

- 22 boxes of m
- 12 packets of
- 80 packets of

a) Display the a on a 1 by 3 m

b) Show the ven are 20 Ngwee and 30 Ngwe

c) Find the matr

Revision exercises

1 Give the order of each matrix.

a) $\begin{bmatrix} 3 & 7 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 & 5 \end{bmatrix}$ c) $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

d) $\begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$ e) $\begin{bmatrix} 2 & 5 & 5 \\ 9 & 2 & 3 \end{bmatrix}$ f) $\begin{bmatrix} 1 & 0 & 2 \\ 4 & 2 & 5 \\ 6 & 7 & 2 \end{bmatrix}$

2 Transpose each matrix in question 1.

3 Solve the following equations for x and y.

a) $\begin{bmatrix} 3x & 0 \\ 5 & 2y \end{bmatrix} = \begin{bmatrix} 12 & 0 \\ 5 & 10 \end{bmatrix}$ b) $\begin{bmatrix} 5 & \frac{1}{2}y \\ x+2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 8 & 2 \end{bmatrix}$

4 Find the product of the two matrices.

a) $\begin{bmatrix} 7 & 2 \\ 3 & 5 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ b) $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 6 & 8 \\ 4 & 7 & 7 \end{bmatrix}\begin{bmatrix} 2 & 5 & 9 \\ 7 & 1 & 6 \\ 5 & 4 & 2 \end{bmatrix}$

5 If $M = \begin{bmatrix} 7 & 12 \\ 10 & 4 \end{bmatrix}$ and $N = \begin{bmatrix} 14 & 12 \\ 10 & -4 \end{bmatrix}$, find the following:

a) MN b) NM c) M^2 d) N^2

6 Explain why matrices cannot be used to solve a system of linear equations if the determinant of the coefficients equals 0.

7 Find the inverse of each matrix.

a) $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 1 \\ 4 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ d) $\begin{bmatrix} -3 & 1 \\ -5 & 3 \end{bmatrix}$

8 The following matrices are singular. Find the values of x for each matrix.

a) $\begin{bmatrix} x & 1 \\ 4 & x \end{bmatrix}$ b) $\begin{bmatrix} -x & -2\frac{1}{2} \\ 4 & x \end{bmatrix}$

c) $\begin{bmatrix} x-2 & 7 \\ 5 & x \end{bmatrix}$ d) $\begin{bmatrix} 2 & 8-x \\ x & 6 \end{bmatrix}$

9 Use a matrix method to solve the following simultaneous equations.

a) $p + 5q = 12$
$3p - 2q = 2$

b) $3m + 2n = 10$
$m + 2n = 4$

c) $4x + 3y = -13$
$-10x - 2y = 5$

10 A street vendor had the following items in his display tray:

- 22 boxes of matches
- 12 packets of potato chips
- 80 packets of chewing gum

a) Display the above information on a 1 by 3 matrix, M.

b) Show the vendor's prices on a 3 by 1 matrix (P) if his prices for the items are 20 Ngwee for a box of matches, 50 Ngwee for a packet of potato chips and 30 Ngwee for a packet of chewing gum.

c) Find the matrix product MP and state what it represents.

Revision and assessment (continued)

11 Musonda invested K4 000.00. He invests part of the money in a savings account and the rest of the money in a fixed deposit account. The savings account gives 6.5% interest per year. The fixed deposit yields 8.0% after one year. The interest earned after one year on the two investments together is K297.50.

a) Represent the situation using a system of linear equations.
b) Write a matrix equation to represent the situation.
c) Solve the equation to find how much Musonda invested in each type of account.

12 Musonda would like to increase the interest he receives in one year to a minimum of K310. The interest rates for the two accounts remain the same. Adapt the calculations in question 11 to find how much he should invest in each type of account so that he receives the desired amount of interest.

13 The bank tells Musonda that the interest rate on the savings account has changed to 7.0%. Musonda makes a change in the investment and he still earns a total of K297.50 in interest. Calculate how much he has now invested in each account.

Assessment exercises

1 Transpose each matrix.

a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 3 & 3 \\ 4 & 5 \end{bmatrix}$ c) $\begin{bmatrix} -3 & 1 \\ -5 & 3 \end{bmatrix}$ d) $\begin{bmatrix} -3 & -2 \\ 5 & 3 \end{bmatrix}$

2 Find the determinant of each matrix in question 1.

3 Find the adjoint of each matrix in question 1.

4 Find the inverse of each matrix in question 1.

5 If $P = \begin{bmatrix} 3 & 2 \\ -2 & 15 \end{bmatrix}$ and $Q = \begin{bmatrix} 3 & 2 \\ 2 & 15 \end{bmatrix}$, find the following.

a) PQ b) QP c) P^2 d) Q^2

6 If $A = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 5 & 6 \\ 4 & 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 & 4 \\ 2 & 6 & 8 \\ 3 & 3 & 5 \end{bmatrix}$ find the following.

a) AB b) BA c) A^2 d) B^2

7 If $M = \begin{bmatrix} p-2 & 3 \\ 5 & p \end{bmatrix}$ and $N = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, use the equation $MN = \begin{bmatrix} -6 & 6 \\ 10 & -2 \end{bmatrix}$ to solve for p.

8 Solve the following equations for x and y.

a) $\begin{bmatrix} x+y & 3 \\ 4 & 2x-y \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 4 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 2x+y & 4 \\ 3x-y & 2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix}$

9 Use Cramer's rule to solve the following equation for x and y.

$\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$

10 Mrs Manda bu
2 kg oranges a
pays K79.00. L
kilogram of to
a) Use the
information
about the
amounts of
oranges an
tomatoes th
women bou
and the tot
each spent
write a syst
equations.
b) Write the sy
in matrix fo
(AX = B).
c) Use Cramer
kilogram of

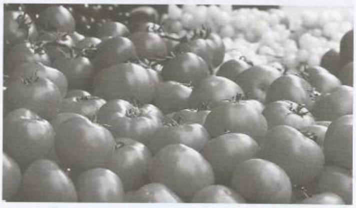
11 On 29 August
football team
before this en
information a

Zambia
Tunisia

In the encoun
a) If FIFA awa
a team that
showing th
b) Draw up a
the two tea
c) Draw up a
show the in
d) Find the ma
two teams

10 Mrs Manda buys 4 kg oranges and 6 kg tomatoes and Mrs Phiri buys 2 kg oranges and 5 kg tomatoes. Mrs Manda pays K125.50 and Mrs Phiri pays K79.00. Let the cost of a kilogram of oranges be Kx and the cost of a kilogram of tomatoes be Ky.

a) Use the information about the amounts of oranges and tomatoes the women bought and the total each spent to write a system of equations.


Tomatoes

b) Write the system in matrix form (AX = B).

c) Use Cramer's rule (or use inverse matrices) to calculate the cost of a kilogram of oranges and a kilogram of tomatoes.

11 On 29 August 1989, the Chipolopolo boys played the Tunisia national football team in the final round of the World Cup qualifiers. Twelve days before this encounter, the Zambia *Daily Mail* published the following information about the standings of the two teams:

	Played	Won	Drawn	Lost
Zambia	5	3	0	2
Tunisia	5	2	1	2

In the encounter, Zambia lost 1–0 to Tunisia.

a) If FIFA awards two points for a win, one point for a draw and no points to a team that loses, write down the given order in a column matrix P showing this allocation of points by FIFA.

b) Draw up a similar table to the one shown above showing the standing of the two teams after the match Zambia lost to Tunisia.

c) Draw up a 2 by 3 matrix M with the headings Won, Drawn, and Lost, to show the information you have worked out for question 11(b).

d) Find the matrix product MP, and give the difference in points between the two teams at this stage.

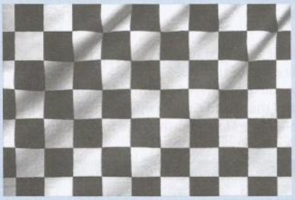
TOPIC 5

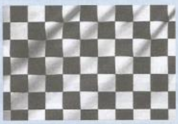
Similarity and congruency

Sub-topic	Specific Outcomes
Application of ratio and proportion	• Calculate the scale on a map. • Calculate length and area using a given scale and calculate a given scale using length and area.
Area and volume of similar figures	• Calculate areas and volumes of similar figures. • Apply ratio and proportion to solve problems of similarity and congruency.

Starter activity

Below is a chequered flag shown in two different sizes. Work in pairs.





1 Measure the two flags (in cm) and complete a copy of the table.

Dimensions	Length	Breadth	Breadth : length	Ratio: $\frac{\text{breadth}}{\text{length}}$
Large flag				
Small flag				

2 Why can we say that the two pictures are similar?
Give reasons for your answer.

3 Which statements are true and which are false?

a) All rectangles are similar.

b) All squares are similar.

c) All circles are similar.

SUB-TOPIC 1 Ap
pro

Introduction

Last year you learnt th
the same shape, but th
same size. If two shape
the angles in the one s
corresponding angles i
The sides of similar s
This means that the ra
constant. For example,
chequered flag on page
of the sides of the smal
corresponding sides of
3 : 5. This ratio can als

Calculate the

A map is a picture that
make maps meaningful
distance a unit length o
given either as a numbe
A number scale is ex
for example, a number
multiplied by 10 000 u
1 mm represents 10 000
The ratio $\frac{\text{distance on}}{\text{corresponding dista}}$
(or scale factor) of the m
you must make sure tha
the same.
A bar scale compares
dimensions of a map, y
of its bar scale. Convert
scale factor.
Below are two examples

1 cm represents 10 km,
the scale is 1 : 1 000 00

SUB-TOPIC 1

Application of ratio and proportion

Introduction

Last year you learnt that similar figures have the same shape, but they do not have to be the same size. If two shapes or objects are similar, the angles in the one shape equal the corresponding angles in the second shape.

The sides of similar shapes are in proportion. This means that the ratio of their sides is constant. For example, the two diagrams of the chequered flag on page 68 are similar. The ratio of the sides of the small flag to the corresponding sides of the large flag is $\frac{3}{5}$ or 3 : 5. This ratio can also be called a scale.

> **Remember**
>
> Any flat shape that you can cut from a piece of paper is a two-dimensional (2D) **shape**. In other words, a 2D shape is a closed shape. This means that if you trace along the edge of the shape, you will end where you started. A shape has length, breadth (or a radius) and area.
>
> An **object** takes up space and has length, breadth and volume. Its faces are shapes.

Calculate the scale on a map

A map is a picture that is similar to the area on the ground that it represents. To make maps meaningful, map makers incorporate a scale. The scale tells us the distance a unit length on the map represents on the ground. The scale can be given either as a number scale or as a bar scale on a map.

A number scale is expressed in the form $a : b$ where a and b are numbers, for example, a number scale of 1 : 10 000 means 1 unit on the map must be multiplied by 10 000 units to get the real distance on the ground. In this way, 1 mm represents 10 000 mm or 1 cm represents 10 000 cm on the ground. The ratio $\frac{\text{distance on the map}}{\text{corresponding distance on the ground}}$ is called the representative fraction (RF) (or scale factor) of the map. As a ratio compares quantities in the same units, you must make sure that the units of the numerator and the denominator are the same.

A bar scale compares lengths measured in different units. If you double the dimensions of a map, you must also double the dimensions on line segments of its bar scale. Convert a bar scale into a number scale before you can find the scale factor.

Below are two examples of bar scales.

1 cm represents 10 km, therefore the scale is 1 : 1 000 000.

1 cm represents 50 km, therefore the scale is 1 : 5 000 000.

Worked example 1

The scale on the map below is 1 : 10 000 000. This means that 1 cm on the map represents 100 km (100 000 cm is 1 km) on the ground. Use the measurement of the bar scale on the map to answer the questions.

1 What length in centimetres on the map represents 200 km on the ground?

2 What distance in kilometres on the ground does 3 cm on the map represent?

3 If Ndola in the Copperbelt is about 600 km from Lusaka as the crow flies, what is the distance between them on the map?

New word

as the crow flies: shortest distance between two places

Worked exampl

Answers

1 1 km = 100 00

The scale is 1:

distance on the
distance on the g

distance on the

Therefore, 200

2 distance on the
distance on the g

Therefore, dista

Therefore, 3 cm

3 From the above
is 600 km from

Activity 1

1 Use the map of Z

a) Measure the d

b) Use the scale t
in kilometres.

2 Kasama is about
the map?

3 Measure the dista
and Zambezi in N
difference betwee

4 A map gives a rep
lengths:

a) a stream that

b) a road on the

5 The scale of a ma

a) the actual len

b) the actual bre

c) the actual dist
the map.

New word

the crow flies: shortest distance between two places

Worked example 1 (continued)

Answers

1 1 km = 100 000 cm; therefore, 200 km = 20 000 000 cm

The scale is 1: 10 000 000

$$\frac{\text{distance on the map}}{\text{distance on the ground}} = \frac{1}{10\,000\,000} \times \frac{2}{2} = \frac{2}{20\,000\,000} \text{ or:}$$

$$\text{distance on the map} = \frac{1}{10\,000\,000} \times \text{distance on the ground}$$

$$= \frac{1}{10\,000\,000} \times 20\,000\,000$$

$$= 2 \text{ cm}$$

Therefore, 200 km on the ground is represented by 2 cm on the map.

2 $$\frac{\text{distance on the map}}{\text{distance on the ground}} = \frac{1}{10\,000\,000}$$

Therefore, distance on the ground $= \frac{10\,000\,000}{1} \times$ distance on the map

$$= 10\,000\,000 \times 3 \text{ cm}$$

$$= 30\,000\,000 \text{ cm}$$

$$= 300 \text{ km}$$

Therefore, 3 cm on the map represents 300 km on the ground.

3 From the above, we can deduce that 100 km is represented by 1 cm. Ndola is 600 km from Lusaka. Therefore, on the map the distance is about 6 cm.

Activity 1

1 Use the map of Zambia on page 70. All distances are measured as the crow flies.
   a) Measure the distance between Kitwe and Lusaka in centimetres.
   b) Use the scale to give the approximate distance between Lusaka and Kitwe in kilometres.

2 Kasama is about 660 km from Lusaka. How many centimetres will this be on the map?

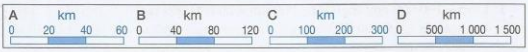
3 Measure the distance on the map between Livingstone in Southern province and Zambezi in North-Western province and use the scale to find the difference between the two towns in kilometres.

4 A map gives a representative fraction of $\frac{1}{20\,000}$. Calculate the following lengths:
   a) a stream that is 90.5 cm long on the map in kilometres
   b) a road on the map that represents 7 km on the ground in centimetres

5 The scale of a map is 1: 50 000. Calculate the following lengths in kilometres.
   a) the actual length of a field measuring 4 cm on the map
   b) the actual breadth of a field measuring 2 cm on the map
   c) the actual distance between two landmarks that are 3 cm apart on the map.

Activity 1 (continued)

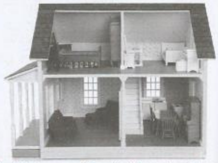
6 Look at the bar scales below and answer the questions that follow.



a) Change each bar scale to a number scale.
b) Give the representative fraction for each scale.

Calculate length and area using a given scale

You can use a **number scale** to calculate lengths and areas. For example, many people build miniatures (such as doll's houses, furniture and cars) as a hobby.



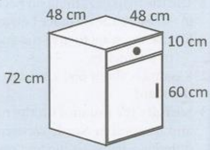
Doll's house with miniature furniture

New word

number scale: a scale expressed in the form $a : b$ where a and b are numbers

Worked example 2

Katele wants to build a kitchen cupboard for a doll's house. The diagram below shows a kitchen cupboard. The doll's house cupboard is built to a scale of 1 : 12.



1 Write down the following measurements for the miniature cupboard:
   a) the side panel b) the drawer front
   c) the door d) the top
2 Calculate the area of the top of the real cupboard.
3 Calculate the area of the top of the miniature cupboard.

Answers

1 a) height: $72 = 1 : 12$ or $\frac{\text{side panel}}{72} = \frac{1}{12}$

therefore, side panel height $= \frac{1}{12} \times 72 = 6$ cm

width: $48 = 1 : 12$ or $\frac{\text{side panel}}{48} = \frac{2}{2}$

therefore, side panel width $= \frac{1}{12} \times 48 = 4$ cm

b) front drawer width $= \frac{1}{12} \times 48 = 4$ cm

front drawer height $= \frac{1}{12} \times 10 = \frac{5}{6}$ cm

Worked exam

c) door hei
door wid
d) top widt
2 Area of the
3 Area of the

Activity 2

1 A map has re
a) the actual
b) the actual
c) the area o
(A = lengt
d) the area o
2 Musa built a
below using h

	Descript
a)	Length o
b)	Length o
c)	Height of
d)	Diameter
e)	Distance

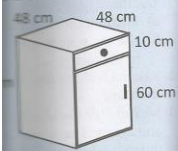
ns that follow.

D	km
0	500 1 000 1 500

a given scale

ouse with miniature furniture

house. The diagram below
is built to a scale of 1 : 12.



Worked example 2 (continued)

c) door height = $\frac{1}{12} \times 60 = 5$ cm

door width = $\frac{1}{12} \times 48 = 4$ cm

d) top width and length = $\frac{1}{12} \times 48 = 4$ cm

2 Area of the top of the real cupboard: 48 cm × 48 cm = 2 304 cm^2

3 Area of the top of the miniature cupboard: 4 cm × 4 cm = 16 cm^2

Activity 2

1 A map has representative fraction of $\frac{1}{20\,000}$. Find the following.

a) the actual length of a field that is 4 cm long on the map, in metres (m)

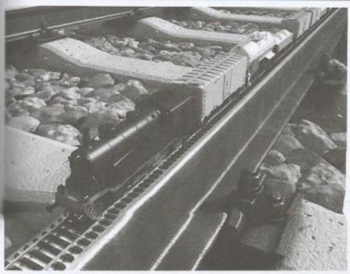
b) the actual breadth of a field that is 2 cm wide on the map, in metres (m)

c) the area of the field on the map, in square centimetres (cm^2)
(A = length × breadth)

d) the area of the field on the ground, in square metres (m^2)

2 Musa built a model train to a scale of 1 : 25. Complete a copy of the table below using his scale.

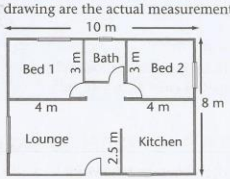
	Description of feature	On the real train	On the model
a)	Length of engine		16 cm
b)	Length of train	20 m	
c)	Height of train		12 cm
d)	Diameter of wheels	1.25 m	
e)	Distance between wheels		6 cm



The model train's track is set on one rail of the track for a full size train. Compare the size of the bolt that ties the rail down with the size of the model train.

Activity 2 (continued)

3 Look at the plan of the house in the diagram below. The measurements on the drawing are the actual measurements of the house.

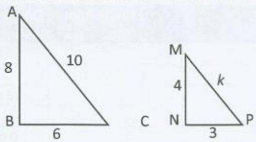


a) If the drawing has a length 5 cm and a breadth 4 cm, calculate the scale.
b) The width of a door in the actual house is 90 cm. What will the width of a door be on the drawing?

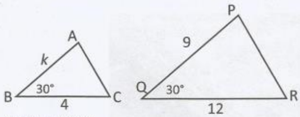
4 If 10 m on the actual house is represented by 10 cm on the drawing, what is the scale?

5 The triangles in each pair are similar. Give the ratio of the corresponding sides in the two triangles and then find the value of k. Use the symbol ||| for similar.

a)



b)



c) △ABC ||| △APQ

SUB-TOPIC 2 Are fig

Calculate the shapes and o

We will first work with

Calculate lengt

We know that the side in proportion. This me the same ratio. For exa then all the sides of th double the length of th shape.

In the diagram:
- The side lengths of side lengths of shap
- The side lengths of the side lengths of s

Worked example

In this example, we questions refer to the with shapes E to H.

1 Find the measure
2 Write down the
3 What conclusion

e measurements on the

m, calculate the scale. What will the width of

n the drawing, what is

f the corresponding sides
the symbol ||| for similar.

SUB-TOPIC 2

Area and volume of similar figures

Calculate the areas and volumes of similar shapes and objects

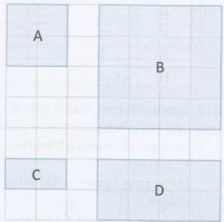
We will first work with similar 2D shapes and then with similar 3D objects (solids).

Calculate length and area using scale

We know that the sides of similar shapes are in proportion. This means that they are in the same ratio. For example, if the ratio is $\frac{1}{2}$, then all the sides of the second shape are double the length of the sides of the first shape.

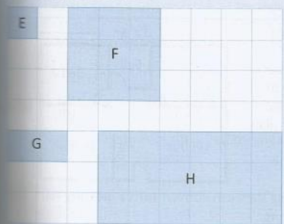
In the diagram:

- The side lengths of shape B are double the side lengths of shape A.
- The side lengths of shape D are double the side lengths of shape C.



Worked example 3

In this example, we investigate the ratio between the areas of the shapes. The questions refer to the diagram above with shapes A to D and the one below with shapes E to H.



1 Find the measurements of shapes A to H.
2 Write down the names of the groups of shapes that are similar.
3 What conclusion can you make about the ratio of the areas of similar shapes?

Worked example 3 (continued)

Answers

1

Shape	Length (units)	Width (units)	Ratio of side lengths	Area (square units)	Ratio of areas
A	2	2	1 : 2	4	1 : 4
B	4	4		16	
C	2	1	1: 2	2	1 : 4
D	4	2		8	
E	1	1	1 : 3	1	1 : 9
F	3	3		9	
G	2	1	1 : 3	2	1 : 9
H	6	3		18	

2 Similar shapes: A and B; C and D; E and F; G and H

3 The ratio between the areas of two similar shapes is the square of the ratio between the sides of the two shapes.

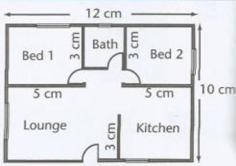
Example:

If the ratio between the sides is 1 : 3 or $\frac{1}{3}$, the ratio between the areas of the shapes is $\left(\frac{1}{3}\right)^2$.

In general, if the ratio between the sides of two similar shapes is $\frac{a}{b}$ or $(a : b)$, the **ratio between the areas** of the shapes is $\left(\frac{a}{b}\right)^2$. So if the sides are increased by a factor of 2, the area will be increased by a factor of $2^2 = 4$.

Activity 3

1 The plan for Phiri's house is drawn to a scale of 1 : 100.



a) Write down the ratio of the floor areas of the plan of the house to the floor areas of the real house.

b) Calculate the area of the floor plan in square centimetres (cm^2).

c) Use the ratio to write down the area of the real house.

2 A church group is given a building they can convert into a meeting hall. The building is 8 m wide and 15 m long.

a) Calculate the area of the floor space of the building.

b) The group decide to extend the building and to double its length and its width. By what factor will the floor space increase?

c) What will the new floor area of the converted building be?

Activity 3 (continued)

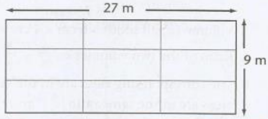
3 A school has a building that is 27 m long and 9 m wide. They decide to convert the building into smaller rooms and to use it as a hostel for students. They divide the building along the length and along the width by 3.

a) Give the ratio of the side length of a smaller room to the side length of the original building.

b) Calculate the area of the floor of the original building.

c) Give the ratio: area of a smaller room : area of the building.

d) Use the ratio to calculate the floor area of the smaller rooms.

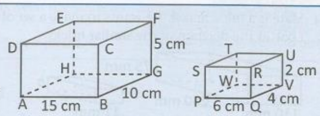


Calculate length, area and volume using scale

In the same way that 2D shapes are similar, 3D objects are similar if their sides are in proportion. This means that corresponding sides are in the same ratio. The next worked example gives you the opportunity to investigate these relationships.

Worked example 4

Look at the drawings of two rectangular prisms (cuboids) and compare the measurements of the sides, the areas of the faces and the volumes of the two solids.



1 Compare the following ratios of the sides of the two solids: AB : PQ; BG : QV and GF : UV. What is your conclusion?

2 Calculate the areas of the faces: ABCD; PQRS; BCFG; QRUV; ABGH and PQVW.

3 Find the ratios $\frac{ABCD}{PQRS}$, $\frac{BCFG}{QRUV}$ and $\frac{ABGH}{PQVW}$. What is your conclusion?

4 Calculate the volume of the two solids in cubic centimetres (cm^3).

5 Find the ratio of the two volumes.

6 Compare the ratios of the sides, the areas of the faces and the volumes? What is your conclusion?

Answers

1 AB : PQ = 15 : 6 = 5 : 2; BG : QV = 10 : 4 = 5: 2; GF : UV = 5 : 2
The ratios of the sides are constant $\left(\frac{5}{2}\right)$; therefore, the sides are in proportion.

2 ABCD = 15 × 5 = 75 cm^2 PQRS = 12 cm^2
BCFG = 50 cm^2 QRUV = 8 cm^2
ABGH = 150 cm^2 PQVW = 24 cm^2

Worked example 4 (continued)

3 $\frac{ABCD}{PQRS} = \frac{75}{12} = \frac{25}{4}$; $\frac{BCFG}{QRUV} = \frac{50}{8} = \frac{25}{4}$ and $\frac{ABGH}{PQVW} = \frac{150}{24} = \frac{25}{4}$

The ratio of the areas of the faces is the square of the ratio of the sides $\left(\frac{5}{2}\right)^2$.

4 Volume (large solid) = length × breadth × height

$= 15 \text{ cm} \times 10 \text{ cm} \times 5 \text{ cm}$

$= 750 \text{ cm}^3$

Volume (small solid) = $6 \text{ cm} \times 4 \text{ cm} \times 2 \text{ cm} = 48 \text{ cm}^3$

5 Ratio of the two volumes = $\frac{750}{48} = \frac{\frac{6}{125}}{\frac{6}{8}} = \frac{125}{8} = \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^3$

6 The corresponding sides are in the same ratio $\left(\frac{5}{2}\right)$, the ratio of the surface areas are in the same ratio $\left(\frac{5}{2}\right)^2$ and the volumes are in the same ratio $\left(\frac{5}{2}\right)^3$.

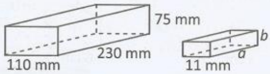
From the above worked example, we can deduce the following.

If two solids are similar:

- their corresponding sides are in the same ratio
- the ratio of their surface areas is equal to (ratio of corresponding sides)^2
- the ratio of their volumes is equal to (ratio of corresponding sides)^3.

Activity 4

1 Mate is a brick maker. He wants to make a set of miniature bricks for his son. Look at the diagram of the similar bricks.



a) Calculate the ratio between the corresponding sides of the bricks.

b) Give the value of the length (a) and the height (b) of the miniature brick.

c) Write down the ratio between the volumes.

d) Convert the measurements of the standard size brick to centimetres (cm) and calculate its volume cubic centimetres (cm^3).

e) Use the ratio between the volumes to find the volume of a miniature brick.

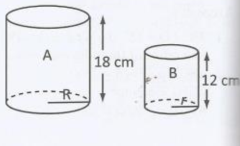
f) Calculate the volume of clay Mate will need to make 1 000 miniature bricks for his son.

2 The two cylinders A and B in the diagram are similar. Use the information on the diagram to find the following.

a) the ratio of the radii of the two bases

b) the ratio of the two curved surface areas

c) the volume of cylinder B if A has a volume of 621 cm^3.



Activity 4 (cont

3 Nosiku's advert
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of 6.6 cm. Calc
a) the height

4 A chocolate ma
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box that is also
diagram, the pr
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a) Calculate th
b) Calculate th
c) The manufa
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that the triangles ar
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Congruent tri

Triangles are congru
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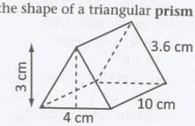
triangle (SSS)
- any two sides and
triangle are equal
- any two angles an
corresponding sid
- the hypotenuse a
and a side of the

Activity 4 (continued)

3 Nosiku's advertising company was asked to make a huge model cola can to place on the roof of a cola distribution centre. They used a scale of 1 : 100. A normal cola can has a volume of 335 ml, a height of 12 cm and a diameter of 6.6 cm. Calculate the following dimensions of the model can.
 a) the height b) the diameter c) the volume.

4 A chocolate manufacturer makes chocolates in the shape of a triangular **prism** (see diagram). The chocolates are packed in a box that is also a triangular prism. In the diagram, the prism is lying on one of its sides. The one triangular face (the base) faces forwards and the other triangular face is at the back.



 a) Calculate the area of a triangular face (the base).
 b) Calculate the volume of the prism (V = area of base × length).
 c) The manufacturer wants to make a box that is similar to, but bigger than the one in the diagram. The ratio of the sides of the bigger box to the sides of the smaller box must be 1.5.
 Give the measurements of the bigger box.
 d) Write down the ratio between the volumes of the two prisms.
 e) Find the volume of the larger prism.

New word

prism: an object with two end faces that are similar and equal, and side faces that are rectangles

Congruent shapes

Shapes are congruent if they are similar and they have the same size. This means that not only must their corresponding angles be equal, but their corresponding sides must be the same length.

When we work with congruent triangles, we use certain conditions to show that the triangles are indeed congruent. You learnt about these conditions in previous grades.

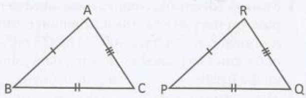
Congruent triangles

Triangles are congruent (≡) if:

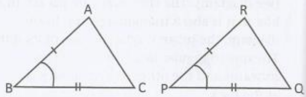
- the sides of one triangle are equal to the corresponding sides of the other triangle (SSS)
- any two sides and the angle between these sides (the included angle) of one triangle are equal to two sides and the included angle of the other triangle (SAS)
- any two angles and one side of one triangle are equal to two angles and the corresponding side of the other triangle (AAS)
- the hypotenuse and a side of a right-angled triangle are equal to the hypotenuse and a side of the other triangle (RHS).

The following diagrams illustrate these conditions.

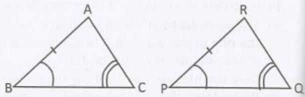
- Side, side, side (SSS)
  In △ABC and △PQR:
  AB = RP
  BC = PQ
  AC = RQ
  Therefore, △ABC ≡ △RPQ.



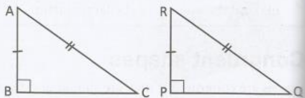
- Side, angle, side (SAS)
  In △ABC and △PQR:
  AB = RP
  BC = PQ
  $A\hat{B}C = R\hat{P}Q$
  Therefore, △ABC ≡ △RPQ.



- Angle, angle, corresponding side (AAS)
  In △ABC and △PQR:
  AB = RP
  $A\hat{B}C = R\hat{P}Q$
  $A\hat{C}B = R\hat{Q}P$
  Therefore, △ABC ≡ △RPQ.



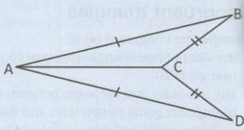
- Right angle, hypotenuse, side (RHS)
  In △ABC and △PQR:
  $A\hat{B}C = R\hat{P}Q = 90°$
  AB = RP
  AC = RQ
  Therefore, △ABC ≡ △RPQ.



Worked example 5

Use the information in the diagram where AB = AD and BC = CD to prove that $A\hat{B}C = A\hat{D}C$.

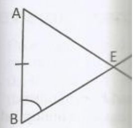
Proof



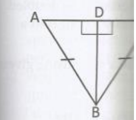
In △ABC and △ADC:
AB = AD (Given)
BC = CD (Given)
AC is a common side
Therefore, △ABC ≡ △ADC (SSS)
Hence, $A\hat{B}C = A\hat{D}C$

Activity 5

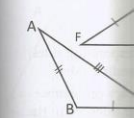
1 In the diagram, AB



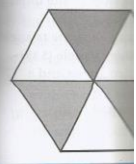
2 The pairs of triangl
SSS or AAS) accordi
a) △ABD and △BCI



c) △ABC and △DEF

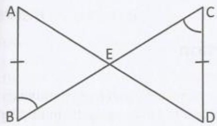


3 A congregation colle
The window is in th
length). Show that t



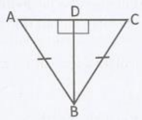
Activity 5

1 In the diagram, AB = DC and $A\hat{B}C = D\hat{C}E$. Prove that $\triangle ABE \equiv \triangle DEC$.

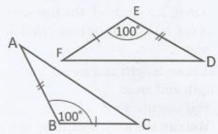


2 The pairs of triangles below are congruent. State the property (for example, SSS or AAS) according to which each pair is congruent.

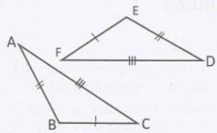
a) $\triangle ABD$ and $\triangle BCD$



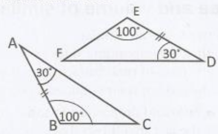
b) $\triangle ABC$ and $\triangle DEF$



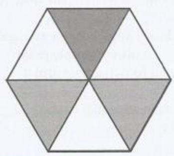
c) $\triangle ABC$ and $\triangle DEF$



d) $\triangle ABC$ and $\triangle DEF$



3 A congregation collected money for a stained glass window for their church. The window is in the shape of a regular hexagon (all sides have the same length). Show that the window is made up of six congruent glass panels.



TOPIC 5

Summary, revision and assessment

Summary

Application of ratio and proportion

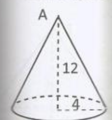
- Calculate the scale on a map:
  - » A number scale is expressed in the form $a : b$ where a and b are numbers.
  - » A number scale of 1 : 10 000 means 1 unit on the map has to be multiplied by 10 000 units to get the real distance on the ground.
  - » The ratio $\frac{\text{distance on the map}}{\text{corresponding distance on the ground}}$ is known as the representative fraction (RF) (or scale factor) of a map.
  - » A bar scale compares lengths measured in different units. If the dimensions of a map are doubled, the line segments of its bar scale must also be doubled.
  - » A bar scale must be converted into a number scale before you can find a scale factor.
- Calculate length and area using a given scale and calculate a scale using given length and area:
  - » You can use a number scale to create similar shapes.
  - » You can use a number scale and similar shapes to calculate lengths.

Area and volume of similar figures

- If two solids are similar:
  - » their corresponding sides are in the same ratio
  - » the ratio of their surface areas is equal to (ratio of corresponding sides)^2
  - » the ratio of their volumes is equal to (ratio of corresponding sides)^3
- Use ratio and proportion to solve similarity and congruency problems.
  Use the fact that if two shapes or objects are similar, their sides are in the ratio $(a : b)$, then their areas are in the ratio $(a : b)^2$, and their volumes are in the ratio $(a : b)^3$.
  Triangles are congruent if:
  - » the sides of one triangle are equal to the corresponding sides of the other triangle (SSS)
  - » any two sides and the angle between them (included angle) of one triangle are equal to two sides and the included angle of the other triangle (SAS)
  - » any two angles and a side of one triangle are equal to two angles and a corresponding side of the other triangle (AAS)
  - » the hypotenuse and a side of a right-angled triangle are equal to the hypotenuse and a side of the other triangle (RHS).

Revision exe

1 On a map of Afric
capital cities of th
the bar scale is 1
the number scale
a) Luanda in Ang
about 30 mm
Calculate the
between them
b) On the map, t
between Lusak
is about 58 m
approximate d
two cities in k
c) Windhoek (Na
approximately
Lusaka as the
the distance b
2 Which of the foll



3 A boy is 140 cm t
is 1 : 20. Find the
a) the height of t
b) the actual heig
c) the height on t
4 The diagram show
of 30 cm and wat
a) Calculate the r
to the height o
b) Calculate the r
the capacity of

Revision exercises

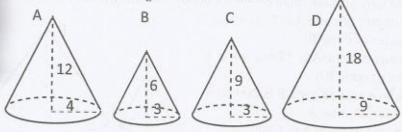
1 On a map of Africa that includes the capital cities of the different countries, the bar scale is 1 cm equals 400 km and the number scale is 1 : 51 400 000.

a) Luanda in Angola and Lusaka are about 30 mm apart on the map. Calculate the approximate distance between them as the crow flies.

b) On the map, the straight distance between Lusaka and Nairobi (Kenya) is about 58 mm. Calculate the approximate distance between the two cities in kilometres.

c) Windhoek (Namibia) is approximately 1 234 km from Lusaka as the crow flies. Calculate the distance between them on the map.

2 Which of the following cones are similar?



3 A boy is 140 cm tall. The ratio of his height on a photograph to his real height is 1 : 20. Find the following in centimetres.

a) the height of the boy on the photograph

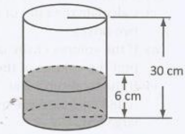
b) the actual height of his little sister who is 3.5 cm tall on the photograph

c) the height on the photograph of a tree that is 2.2 m tall in real life

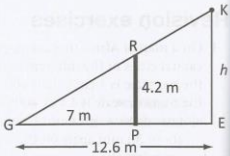
4 The diagram shows a cylinder with a height of 30 cm and water to a depth of 6 cm.

a) Calculate the ratio of the depth of the water to the height of the cylinder.

b) Calculate the ratio of the volume of water to the capacity of the cylinder.

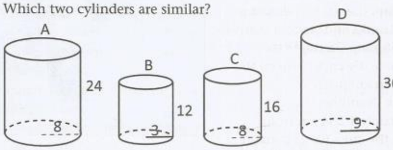


5 In the diagram, K is a kite that is tied to the ground at G by a rope GK. The rope just passes over a vertical pole PR of height 4.2 m. Given that the kite is h metres vertically above E, GE = 12.6 m and GP = 7 cm, calculate the value of h.



Assessment exercises

1 Which two cylinders are similar?



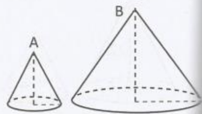
2 In the diagram, the diameter of cone A is $\frac{1}{10}$ of the diameter of cone B. Given that the cones are similar, answer the following questions.

a) If the diameter of cone A is 5 cm, find the diameter of cone B.

b) If the radius of cone A is 12 cm, what is the radius of cone B?

c) If the surface area of cone B is 200 cm^2, find the area of cone A.

d) If the volume of cone A is 1 000 cm^3, find the volume of cone B.



3 KOOL drink is sold as a powder in large and small boxes as shown below. The boxes are similar.

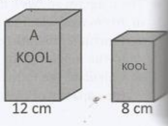
a) Find the ratio of corresponding sides of the two boxes.

b) Calculate the ratio of the surface areas of the two boxes.

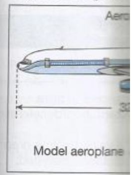
c) Calculate the ratio of the volumes of the two boxes.

d) If the volume of the large box is 1 728 cm^3, find the volume of the small box.

e) If the total surface area of the small box is 488 cm^2, find the surface area of the large box.



4 The diagram shows
length of aeroplane

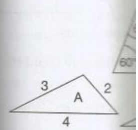


a) Calculate the len
length of the aer

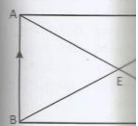
b) Calculate the rati
of the model aer

c) Calculate the vol
aeroplane is 640

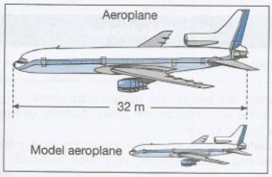
5 Which pairs of trian



6 Below is the diagram
rectangle. Prove that
$\triangle$BEC are also congr



4 The diagram shows an aeroplane and a model of the aeroplane. The ratio, length of aeroplane to length of the model is 80 : 1.

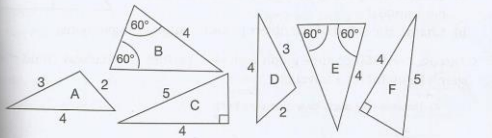


a) Calculate the length of the model aeroplane in metres, given that the length of the aeroplane is 32 m.

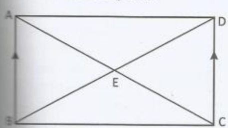
b) Calculate the ratio of the surface area of the aeroplane to the surface area of the model aeroplane.

c) Calculate the volume of space in the model aeroplane if the volume of the aeroplane is 640 m^3.

5 Which pairs of triangles are congruent?



6 Below is the diagram of a gate. The outer shape of the gate, ABCD, is a rectangle. Prove that △ABC and △DCB are congruent and that △AED and △BEC are also congruent.

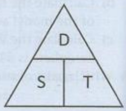


TOPIC
6 Travel graphs

Sub-topic	Specific Outcomes
Distance–time graphs	• Compute average speed, distance and time.
Velocity–time graphs	• Compute average speed, distance and time. • Determine acceleration and retardation/deceleration. • Draw travel graphs. • Calculate the distance under a velocity–time graph. • Relate area under a graph to distance travelled.

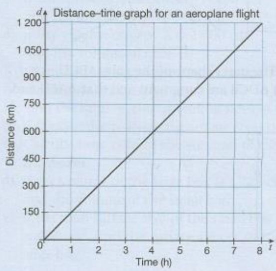
Starter activity

1 On the triangle alongside, D refers to distance, S refers to speed and T refers to time. How do you think you can use this information? Work with a partner.



2 An object covers 800 m (metres) in 32 s (seconds).
   a) What is the speed of the object in m/s (metres per second)?
   b) Change the speed of the object to km/h (kilometres per hour).

3 Discuss the distance–time graph with your partner. In what way could the graph be useful for a traveller?



SUB-TOPIC 1 D

A travel graph is a li
that gives a picture
object moves. The m
can be on foot, on a
by car, bus, train, ae
and so on.
You can show ho
distance (or displace
changes over time o
distance–time graph,
how the speed (or ve
changes over time o
speed–time (or veloc
time) graph.
You must know h
first revise how to ca
known.

Compute av

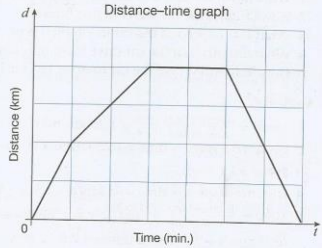
The relationship betw
Use the triangle to he
- speed, cover S so t
- time, cover T so th
- distance, cover D s

Note:
In the above formula
- The units you use
  sign. For example,
  be in metres and th
- Units can be in kil
  metres (m), secon
- You could change
  metres per second
- If the speed is no
  average speed =

SUB-TOPIC 1 Distance–time graphs

A travel graph is a line graph that gives a picture of how an object moves. The movement can be on foot, on a bicycle, by car, bus, train, aeroplane, and so on.

You can show how the distance (or displacement) changes over time on a distance–time graph, and how the speed (or velocity) changes over time on a speed–time (or velocity–time) graph.

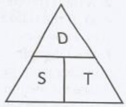


You must know how to draw as well as interpret a distance–time graph. We will first revise how to calculate speed, distance or time if the other two quantities are known.

Compute average speed, distance and time

The relationship between distance, time and speed is given by the formulae below. Use the triangle to help you find:

- speed, cover S so that $S = \frac{D}{T}$; speed = $\frac{\text{distance}}{\text{time}}$
- time, cover T so that $T = \frac{D}{S}$; time = $\frac{\text{distance}}{\text{speed}}$
- distance, cover D so that $D = S \times T$; distance = speed × time



Note:

In the above formulae, the speed is constant (it does not change).

- The units you use in the formulae must be the same on both sides of the equal sign. For example, if the speed is in metres per second (m/s), the distance must be in metres and the time in seconds.
- Units can be in kilometres (km), hours (h), kilometres per hour (km/h) or metres (m), seconds (s) or metres per second (m/s).
- You could change units from, for example, kilometres per minute (km/min.) or metres per second (m/s) to kilometres per hour (km/h).
- If the speed is not constant, you can calculate the average speed as follows:

  average speed = $\frac{\text{total distance covered}}{\text{total time taken}}$.

Worked example 1

A car drives 5 km in 4 minutes.

1 Write down the speed of the car in kilometres per minute (km/min.).
2 How far does the car travel in one hour?
3 What is the speed of the car in kilometres per hour (km/h)?
4 What distance can the car drive in 90 minutes?
5 How long will it take the car to drive 165 km?

Answers

1 Speed = $\frac{\text{distance (km)}}{\text{time (min.)}} = \frac{5}{4} = 1.25$ km/min.
2 Distance = speed × time = 1.25 km/m × 60 m = 75 km
3 Speed = 75 km/h
4 Distance = speed × time = 75 km/h × 1.5 h = 112.5 km
5 Time = $\frac{\text{distance (km)}}{\text{speed(km/s)}} = \frac{165 \text{ km}}{75 \text{ km/h}} = 2.2$ hours
or, 2 hours + $\frac{2}{10}$ of 60 minutes = 2 h 12 min.

Activity 1

1 A man walks eight kilometres in two hours.
  a) What is his speed in kilometres per hour (km/h)?
  b) Walking at the same speed, how long will it take him to walk 1 km?
  c) Walking at the same speed, how far can he walk in 45 minutes?
2 A train travels at an average speed of 100 km/h.
  a) If the train travels for one and a half hours, what distance does it cover?
  b) How long does the train take to cover 50 km?
3 The Olympic swimmer Chad le Clos won the 200 m butterfly race for men in 1 min. 52.96 s at the London Olympics in 2012.
  a) Calculate his speed in metres per second (m/s).
  b) Convert his swimming speed to kilometres per hour (km/h).


Swimmer

4 At the same Olympics, Gerald Phiri ran the 100 m semi-final in 10.11 seconds.
  a) Calculate Phiri's speed in metres per second (m/s).
  b) Convert his speed to kilometres per hour (km/h).


Runners in the 100 m race

Scalar and vector quantities

A **scalar** is a quantity that has size but not direction. A quantity that has both size (magnitude) and direction is called a **vector**.

Examples:

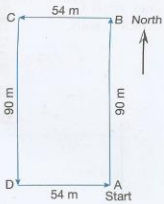
- distance is a scalar, and displacement (for example, 10 m up or down) is a vector
- speed is a scalar, and velocity (for example, 10 m/s to the right or left) is a vector.

New words

scalar: a quantity that has magnitude (size) only
vector: a quantity that has both magnitude (size) and direction

The following symbols are used for vector quantities:

- $\vec{s}$ for displacement
- $\vec{v}$ for velocity
- $\vec{a}$ for acceleration.



We can use distance and displacement to explain the difference between a scalar and a vector. Imagine that a player runs along the edge of a soccer field to get fit. The distance he covers to make one run from starting point A, then to B, C and D and back again to point A is the perimeter of the soccer field.

Distance the runner covers:
90 m + 54 m + 90 m + 54 m
= 288 m

The displacement is a measure of the overall change in the runner's position; in this case, he ends up at the starting point and so the displacement is 0.

However, when the coach's young son runs from point A to point B, his displacement is 90 m, and the distance he covers is 90 m.

Activity 2

Classify each quantity as a scalar or a vector quantity.

1 225 km
2 165 km south
3 600 m/h north
4 9 km
5 333 m/s
6 9.8 m/s downwards
7 4 km/s upwards

Interpreting distance–time graphs

When you interpret a distance–time graph, remember that you are looking at a 'picture' that shows you how long it took to cover a certain distance.

Important facts to remember:

- Speed is given by the distance covered in a unit of time (for example, in a second, a minute or an hour). Example: If you covered 120 km in three hours, your speed was $\frac{120\text{ km}}{3\text{ h}}$ = 40 km/h.

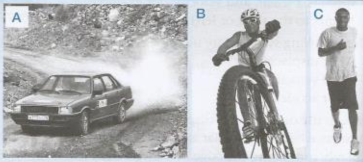
- The gradient between any two points on a line is always given by

$$\frac{\text{change in vertical distance}}{\text{change in horizontal distance}} \text{ or } \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$$

- The gradient of a line on a displacement–time graph represents the speed or velocity of the movement.

Worked example 2

The graph below shows the motion of car A, cyclist B and jogger C. Work in pairs.



Motion of car A, cyclist B and jogger C

Worked exam

Answers

1 They all cov
on the verti
2 The jogger t
3 The car had
4 Vectors are r

distances and ti

Worked exam

The graph below

150

Worked example 2 (continued)

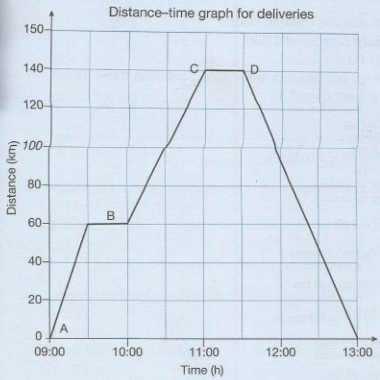
Answers

1. They all covered the same distance (the graphs all end at the same height on the vertical axis).
2. The jogger took the longest time to jog the distance.
3. The car had the highest speed as it drove the distance in the shortest time.
4. Vectors are not involved as no information about directions was given.

If distances and times are given, you can calculate the speed of an object.

Worked example 3

The graph below describes a return journey Mate took by car to make deliveries.



1. How far did Mate go before he started his return journey?
2. How far from his starting point did he make the first stop?
3. At what time did he arrive at his first stop?
4. For how long did he stop at the first stop?
5. At what time did he start his return journey?
6. At what time did he arrive back at the starting place?

Worked example 3 (continued)

7 How many stops did he make on the journey?
8 What was the average speed of the car between points A and B?
9 What is the gradient of the line between A and B?
10 What was the average speed of the car when Mate passed point C?
11 What is the gradient of the line between B and D?
12 Compare your answers for questions 8 and 9, and for questions 10 and 11. What do you notice?
13 At what average speed did Mate drive back to the starting point? Round off your answer to the nearest whole number.

Answers

1 140 km
2 He stopped when he was 60 km from the starting point.
3 09:30
4 He stopped for 30 minutes (half an hour).
5 He started the return journey at 11:30.
6 He was back by 13:00.
7 There were two stops (no distance was covered between those times).
8 Average speed $= \frac{\text{distance covered}}{\text{time taken}}$

$= \frac{60 \text{ km}}{30 \text{ min.}}$

$= \frac{60 \text{ km}}{\frac{1}{2} \text{ h}} = 120 \text{ km/h}$

9 Gradient $= \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$

$= \frac{60 \text{ km}}{\frac{1}{2}} = 120$

10 Average speed $= \frac{\text{distance covered}}{\text{time taken}}$

$= \frac{140 \text{ km} - 60 \text{ km}}{1 \text{ h}}$

$= \frac{80 \text{ km}}{1 \text{ h}} = 80 \text{ km/h}$

11 Gradient $= \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$

$= \frac{140 \text{ km} - 60 \text{ km}}{11 \text{ h} - 10 \text{ h}}$

$= \frac{80}{1} = 80 \text{ km/h}$

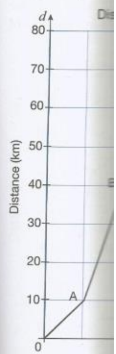
12 The gradient represents the average speed.
13 Average speed $= \frac{\text{distance covered}}{\text{time taken}}$

$= \frac{140 \text{ km}}{1.5 \text{ h}}$

$= 93.3 \text{ km/h} \approx 93 \text{ km/h}$

Activity 3

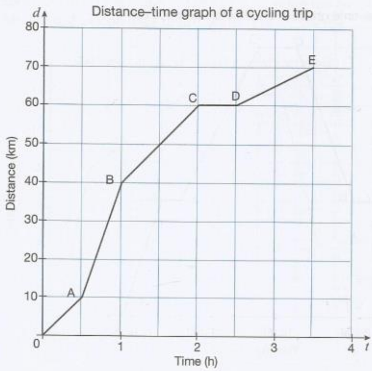
1 The graph below
The journey last



a) What distance
i) 1 h
ii) 2 h
iii) $2\frac{1}{2}$ h
iv) $3\frac{1}{2}$ h
b) How long did t
i) 5 km
ii) 50 km
iii) 10 km
iv) 60 km
c) What is the cy
i) from the st
ii) from A to B
iii) from B to C
iv) from D to E
d) Explain what h
e) Did the cyclist

Activity 3

1 The graph below shows the distance a cyclist rode in a certain time. The journey lasted $3\frac{1}{2}$ hours. The cyclist's trip starts at the origin (0).



a) What distance did the cyclist ride after each length of time?
   i) 1 h
   ii) 2 h
   iii) $2\frac{1}{2}$ h
   iv) $3\frac{1}{2}$ h

b) How long did the cyclist take to cycle each distance?
   i) 5 km
   ii) 50 km
   iii) 10 km
   iv) 60 km

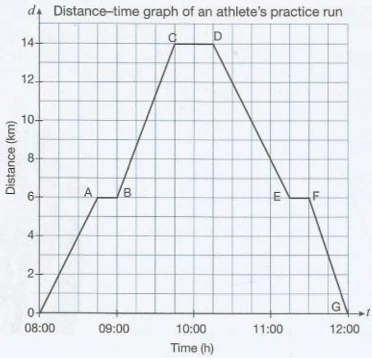
c) What is the cyclist's speed for each distance?
   i) from the start to A
   ii) from A to B
   iii) from B to C
   iv) from D to E

d) Explain what happen between C and D.

e) Did the cyclist cycle back to where his ride started? Explain your answer.

Activity 3 (continued)

2 A long-distance athlete starts running at his home on a practice run as represented in the graph.



a) What distance did the athlete run in total?

b) Briefly describe the athlete's run from the start to point G.

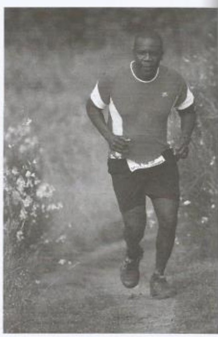
c) How many times and for how long, did the athlete stop running on his practice run?

d) At what speed did the athlete run for each distance?

- **i)** from where he started to A
- **ii)** from B to C
- **iii)** from D to E
- **iv)** from E to G

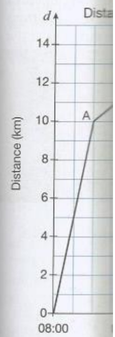
e) What was the athlete's average speed over the whole distance:

- **i)** including rest times
- **ii)** excluding rest times?



Activity 3 (conti

3 Monde cycled to
tyres had a punc
spent the first 15
Monde had to st

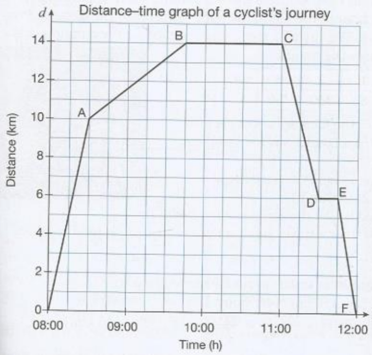


a) At what time
b) How far is it f
c) How long did
fixing the pun
d) How far from
on his return
e) For how long
f) Calculate the
- **i)** from Mon
the punct
- **ii)** while Mo
- **iii)** from Mat
inflate th
- **iv)** on the las

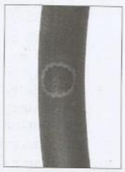
practice run as

Activity 3 (continued)

3 Monde cycled to Mate's house, but 10 km into his trip one of his bicycle's tyres had a puncture. Monde walked the rest of the way, pushing the bike. He spent the first 15 minutes at Mate's house fixing the tyre. On the way home, Monde had to stop once to inflate the tyre.



a) At what time did Monde reach Mate's house?
b) How far is it from his house to Mate's house?
c) How long did Monde stay at Mate's house after fixing the puncture?
d) How far from Mate's house did he inflate the wheel on his return journey?
e) For how long did he stop to inflate the tyre?
f) Calculate the speed during each part of the journey.
   i) from Monde's house until his bike had the puncture
   ii) while Monde walked to Mate's house
   iii) from Mate's house to where Monde stopped to inflate the tyre
   iv) on the last leg of Monde's journey, from E to F (his house)



Patch on a bicycle tyre

Drawing distance–time graphs

We always show the independent variable (time) on the horizontal axis and the other variable (in this case distance or displacement) on the vertical axis.

Worked example 4

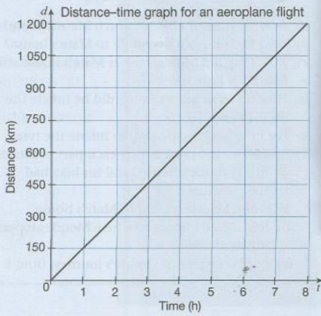
A light aircraft took off from Kenneth Kaunda International Airport in Lusaka. At the end of every hour of the flight, the pilot wrote down the distance from the airport in Lusaka. The aircraft's speed was constant for the duration of the flight. The table shows the results.

Time (h)	1	2	3	4	5
Distance (km)	150	300	450		

1 Are the values in the table scalars or vectors?
2 How far away from Lusaka was the aeroplane after four hours?
3 What was the aeroplane's average speed in kilometres per hour (km/h)?
4 How long would it take the aeroplane to fly 900 km?
5 Use the table to draw a travel graph if the aeroplane flew for five hours.

Answers

1 The values are taken as scalars as no directions are given.
2 The aeroplane flew 150 km every hour: 150 km × 4 = 600 km
3 Speed = $\frac{\text{distance covered}}{\text{time taken}} = \frac{150\text{ km}}{1\text{ h}} = \frac{300\text{ km}}{2\text{ h}} = \frac{450\text{ km}}{3\text{ h}} = 150$ km/h
4 Time taken
$= \frac{\text{distance (km)}}{\text{speed (km/h)}}$
$= \frac{900\text{ km}}{150\text{ km/h}}$
$= 6$ h
5 As time is the independent value, plot it on the horizontal axis and the distance (in kilometres) on the vertical axis. (Always plot the independent value on the horizontal axis.)



Activity 4

1 Kabila left hom
for one hour to
home at an ave
a) Calculate th
b) How long d
c) Draw a trave
d) How far did
2 Mrs Chewe dro
left home at 08
one hour, she i
another hour. S
hours in Lusaka
a) Calculate Mr
b) Draw a dista
c) Use the grap
3 Basiku took a b
The bus left at 1
for 15 minutes.
an average spee
for 15 minutes.
journey in 30 m
a) At what aver
first 80 km?
b) At what time
after the first
c) How far was
it stopped th
d) At what aver
e) Draw a distan
d) Use the graph
4 The flying distan
1 211 km. Johan
and 50 minutes.
a) Find the displ
i) Lusaka to
ii) Johannesb
b) Calculate the
an aeroplane
Johannesbur
c) Draw a displ
d) Are the valu

Activity 4

1 Kabila left home by car at 09:00 and travelled at an average speed of 60 km/h for one hour to attend a meeting. After one and a half hours, he returned home at an average speed of 50 km/h.
   a) Calculate the distance he travelled to his destination.
   b) How long did it take him to get home after the meeting?
   c) Draw a travel graph to represent the situation.
   d) How far did he travel in total?

2 Mrs Chewe drove by car from her town to Lusaka, which is 80 km away. She left home at 08:00 and drove at an average speed of 30 km/h. After driving for one hour, she increased her speed to an average of 50 km/h and drove for another hour. She reached Lusaka at 10:00. After spending two and a half hours in Lusaka, she started driving home at an average speed of 80 km/h.
   a) Calculate Mrs Chewe's average speed on her journey to Lusaka.
   b) Draw a distance–time travel graph to represent the situation.
   c) Use the graph to find out at what time Mrs Chewe arrived home.

3 Basiku took a bus to a destination 200 km from home. The bus left at 10:00. It stopped after 80 km at 11:15 for 15 minutes. Then the bus drove for another hour at an average speed of 70 km/h and then stopped again for 15 minutes. The bus completed the rest of the journey in 30 minutes.

Waiting at the bus stop

   a) At what average speed did the bus drive for the first 80 km?
   b) At what time did the bus start the journey again after the first stop?
   c) How far was the bus from the starting point when it stopped the second time?
   d) At what average speed did the bus cover the last part of the journey?
   e) Draw a distance–time travel graph to represent the situation.
   d) Use the graph to find at what time Basiku arrived at her destination.

4 The flying distance between Lusaka and Johannesburg (South Africa) is 1 211 km. Johannesburg is due south of Lusaka. The flying time is 1 hour and 50 minutes.
   a) Find the displacement of an aeroplane on flight from:
      i) Lusaka to Johannesburg
      ii) Johannesburg to Lusaka.

   b) Calculate the average velocity of an aeroplane on a flight between Johannesburg and Lusaka.
   c) Draw a displacement–time graph to represent the situation.
   d) Are the values you worked with in this problem vector or scalar quantities?

SUB-TOPIC 2 Velocity–time graphs

Velocity–time graphs are similar to distance–time graphs, except that the quantity on the vertical axis is speed (a scalar) or velocity (a vector). In this section, you will start by learning about calculations of velocity and acceleration.

Determine acceleration and deceleration

As you know, average speed = $\frac{\text{distance covered}}{\text{time taken}}$.

If the speed is constant, it means that the same distance is travelled in every time unit. For example, 20 km/h means that in every hour, 20 km are travelled.

It could happen that the speed or velocity increases the further someone travels. The rate at which the velocity changes is called **acceleration**. It is a vector quantity. If the change in the velocity is an increase, then the acceleration is in the same direction as the movement. If it is in the opposite direction to the movement and the object is slowing down, it is called **deceleration** (negative acceleration) or retardation.

We use the equation below to calculate the average acceleration of an object over an interval of time:

average acceleration, $\vec{a} = \frac{\text{change in velocity}}{\text{time}} = \frac{\vec{v_f} - \vec{v_i}}{t_f - t_i}$

where $\vec{v_i}$ is the initial velocity at the start (t_i) of the interval and $\vec{v_f}$ is the final velocity at the end of the time interval (t_f).

New words

acceleration: the rate at which an object's velocity changes
deceleration: negative acceleration (or slowing down)

Worked example 5

The velocity of an object at the beginning of a time interval of five seconds was 20 m/s and at the end of the interval it was 70 m/s. Calculate the acceleration.

Answer

Let the velocity at the start (t_0) of the time interval be $\vec{v_0}$ and at the end (t_5), the velocity is $\vec{v_5}$. The average acceleration is:

$$\vec{a} = \frac{\vec{v_5} - \vec{v_0}}{t_5 - t_0} = \frac{70 \text{ m/s} - 20 \text{ m/s}}{5 \text{ s}} = \frac{50 \text{ m/s}}{5 \text{ s}} = \frac{10 \text{ m/s}}{1 \text{ s}} = 10 \text{ m/s/s} = 10 \text{ m/s}^2.$$

The units of acceleration (velocity/time) are: m/s^2 (or km/h/s, and so on).

Activity 5

1 A car accelerates from 0 km/h to 100 km/h in 7.9 seconds.
  a) Convert the speed of 100 km/h to metres per second (m/s).
  b) Calculate the acceleration of the car in m/s^2.

Activity 5 (conti

2 A train leaves a
accelerates to 2
speed for two h
a) Calculate th
travelled du
b) What is the
accelerated?
c) How far doe
hours at the
3 An aeroplane n
of 280 km/h fo
a) Convert the
off to the ne
b) If the aeropl
seconds wou
c) If the aeropl
it take to rea

Draw travel under such

When working with
velocity, which is lab
which is labelled on
different quantities.
We can find:
- the velocity if the
- the time taken if t
- the acceleration if
- the displacement

Worked exampl

1 A motorbike dr
with a velocity
10 seconds late
a) If the rate o
time interv
b) Draw up a t
velocity at e
motorbike

Activity 5 (continued)

2 A train leaves a station at 08:00 and travels at 30 km/h for 15 minutes. It then accelerates to 2 km/min. for 10 minutes. After that it continues at the new speed for two hours.

a) Calculate the distance the train travelled during the first 15 minutes.
b) What is the velocity after the train accelerated?
c) How far does the train travel in two hours at the final velocity?

3 An aeroplane needs to reach a velocity of 280 km/h for take-off.
a) Convert the velocity needed for take-off to metres per second (m/s) (round off to the nearest whole number).
b) If the aeroplane accelerates from 0 m/s at a rate of 2 m/s^2, how many seconds would it need to reach take-off velocity?
c) If the aeroplane accelerates from rest at 1 m/s^2, how many seconds would it take to reach take-off velocity?

Draw travel graphs and determine the distance under such graphs

When working with velocity–time graphs, the dependent variable is speed or velocity, which is labelled on the vertical axis. The independent variable is time, which is labelled on the horizontal axis. We can use a velocity–time graph to find different quantities.

We can find:

- the velocity if the time taken is known
- the time taken if the velocity is known
- the acceleration if the velocity and the time taken are known
- the displacement if the velocity and the time taken are known.

Worked example 6

1 A motorbike drove in a straight line. It started with a velocity of 0 m/s and passed a lookout point 10 seconds later at a velocity of 20 m/s.

a) If the rate of acceleration was constant over the time interval, calculate the acceleration in m/s^2.
b) Draw up a table and record the size of the velocity at each of the ten seconds the motorbike moved.

Worked example 6 (continued)

c) Use the values in the table and draw a graph to represent the situation.
d) Find the gradient of the line and compare it with your answer in question 1(c).
e) Read off the velocity on your graph after four and a half seconds.
f) Use you graph to find after how many seconds the velocity was 13 m/s.

Answers for number 1

1 a) Acceleration, $\vec{a} = \frac{\vec{v_5} - \vec{v_0}}{t_5 - t_0}$

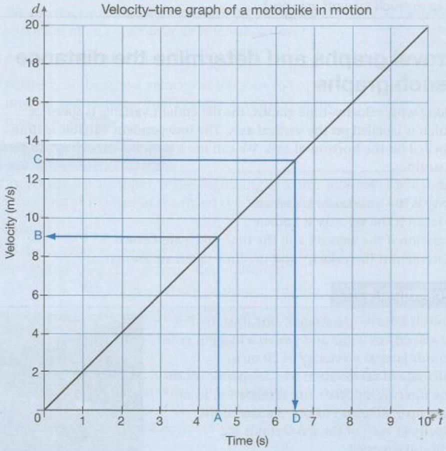
$= \frac{20 \text{ m/s} - 0 \text{ m/s}}{10 \text{ s}}$

$= \frac{20 \text{ m/s}}{10 \text{ s}}$

$= 2 \text{ m/s}^2$

b, c)

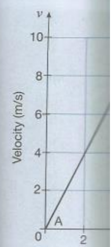
Time (s)	0	1	2	3	4	5	6	7	8	9	10
Velocity (m/s)	0	2	4	6	8	10	12	14	16	18	20



Worked exampl

d) The gradien
gradient be
$m =$ change i
cha
$= \frac{14 \text{ m/s} -}{7 \text{ s} -}$
$= \frac{12 \text{ m/s}}{6 \text{ s}}$
$= 2 \text{ m/s}^2$
e) Start at poin
Draw a verti
draw a horiz
read off the
f) Start on the
a horizontal
the point of
the time axis

2 Look at the grap



a) What happen
i) A and B
iii) C and D
b) Did the motio
c) Calculate the
d) Calculate the

Worked example 6 (continued)

d) The gradient is constant (a straight line). Start by calculating the gradient between 2 s and 7 s.

$$m = \frac{\text{change in } y\text{-values between any two points}}{\text{change in corresponding } x\text{-values}}$$

$$= \frac{14\ \text{m/s} - 2\ \text{m/s}}{7\ \text{s} - 1\ \text{s}}$$

$$= \frac{12\ \text{m/s}}{6\ \text{s}}$$

$$= 2\ \text{m/s}^2\ (\text{or m/s/s})$$

e) Start at point A on the graph, $t = 4\frac{1}{2}$ s.

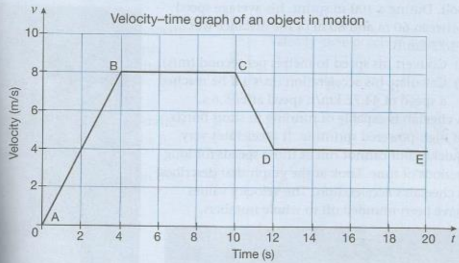
Draw a vertical line to the graph and from the point of intersection, draw a horizontal line to the vertical axis and read off the velocity at point B (the value is 9).

f) Start on the vertical axis at point B (13 m/s). Draw a horizontal line to intersect the graph and from the point of intersection draw a vertical line to the time axis. Read off the value at point D: $6\frac{1}{2}$ s.

Note

The acceleration is positive, therefore the object is accelerating in the direction in which it is moving.

2 Look at the graph that describes the velocity of an object as time passes.



a) What happened between the following points?

i) A and B *ii)* B and C

iii) C and D *iv)* D and E

b) Did the motion end at point E?

c) Calculate the acceleration between points A and B.

d) Calculate the acceleration between points C and D.

Worked example 6 (continued)

Answers for number 2

2 a) i) The object accelerated and its velocity increased from 0 m/s to 8 m/s as time passed.

ii) The object's velocity was constant and no acceleration took place.

iii) The object was subject to negative acceleration or retardation (deceleration). This means that the velocity decreased and the object moved slower.

iv) The object's velocity was constant again and no acceleration took place.

b) The object had velocity at point E, so the motion did not end; this is only the end of the observation process.

c) $\bar{a} = \frac{\bar{v}_5 - \bar{v}_0}{t_5 - t_0} = \frac{8\text{ m/s} - 0\text{ m/s}}{4\text{ s}} = \frac{8\text{ m/s}}{4\text{ s}} = 2\text{ m/s}^2$

d) $\bar{a} = \frac{4\text{ m/s} - 8\text{ m/s}}{(12 - 10)\text{ s}} = \frac{-4\text{ m/s}}{2\text{ s}} = 2\text{ m/s}^2$

Activity 6

1 The fastest human runner on record is Usain Bolt. During a 100-m sprint, his average speed between 60 m and 80 m of the distance was 44.72 km/h.

a) Convert his speed to metres per second (m/s).

b) Calculate his acceleration (m/s^2) if he reached a speed of 44.72 km/h speed after 5.6 s.

2 A cheetah is capable of running in short bursts of high-powered sprinting. It accelerates very quickly, but cannot run at those speeds for long periods of time. Look at the graph that describes a cheetah's acceleration. The velocity values have been rounded off to whole numbers.

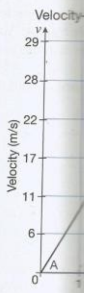
Cheetah

Usain Bolt

Activity 6 (con

a) After how

b) Calculate th



3 The graph show of an object aga

a) What was th the object af seconds?

b) What was th of the object five seconds?

c) Give the acce each time int

i) B to C

ii) C to D

iii) D to E

d) What was the the object aft length of time

i) 25 s

ii) 30 s

s to 8 m/s

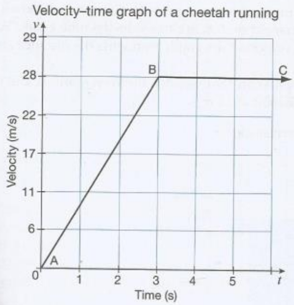
k place.
tion
d the object

ion took

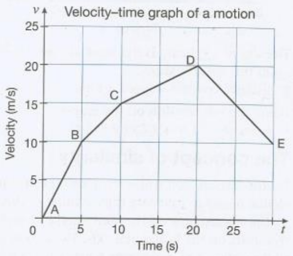
d: this is

Activity 6 (continued)

a) After how many seconds did the cheetah reach its top speed?
b) Calculate the cheetah's acceleration in the first three seconds.



3 The graph shows the velocity of an object against time.
   a) What was the velocity of the object after five seconds?
   b) What was the acceleration of the object over the first five seconds?
   c) Give the acceleration over each time interval.
      i) B to C
      ii) C to D
      iii) D to E
   d) What was the velocity of the object after each length of time?
      i) 25 s
      ii) 30 s

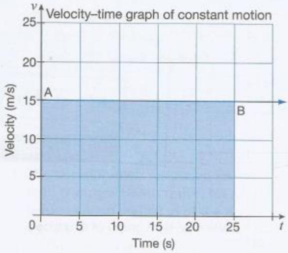


Relate the area under a velocity–time graph to distance travelled

You already know about the relationships between velocity, displacement and time (or speed, distance and time) and between velocity, time and acceleration.

We can also connect displacement or distance to a velocity–time graph. You will find that the area under a velocity–time graph represents the distance an object moved.

The graph shows an object with constant velocity. Between points A and B, the velocity of the object is constant at 15 m/s.



The size of a velocity is the same as speed = $\frac{\text{distance covered}}{\text{time taken}}$

From this we also know:

- distance covered = speed × time

Using the information on the graph:

- distance = 15 m/s × 25 s = 375 m

The concept of similarity

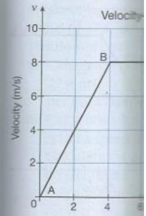
In this section, you will learn about the area under a curve. We use the concept of *similar figures* to calculate the meaning of this area.

The shaded area on the above graph covers three units on the vertical axis and five units on the horizontal axis. The area is 15 square graph units. Each vertical unit on the graph represents 5 m/s and each horizontal unit on the graph represents 5 s. So, the shaded rectangle, which is 3 by 5, represents a larger similar rectangle, which is 15 by 25. We use a similar rectangle (which fits on the graph) to represent the large rectangle. To find the correct area, multiply the units by the correct factor (which we can deduce because the rectangles are similar).

The above example shows that the area under the velocity–time graph gives the distance travelled in the part of the motion at which we are looking.

Worked example

Look at the velocity–



1 Calculate the area
a) B and C
d) D and E

2 Use the area unde travelled in the re

Answers

1 a) Area = vertical
= (8 – 0) ×
= 8 × 6
= 48 squa

b) Area = area of
= $\frac{1}{2}$ × base
= $\frac{1}{2}$ × 4 ×
= 16 squa

c) Area = area of
= 4 × 2 +
= 8 + 4
= 12 squa

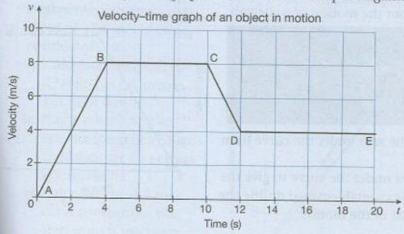
d) Area = (20 – 12
= 8 × 4
= 32 squa

e) Area = 48 + 16

2 The distance the the vertical unit seconds. When yo (m/s × s = m). The

Worked example 7

Look at the velocity–time graph from Worked example 6 again.



1 Calculate the area under the graph between each pair of points.
   a) B and C
   b) A and B
   c) C and D
   d) D and E
   e) A and E

2 Use the area under the graph (A to E) to write the distance the object travelled in the relevant part of the motion.

Answers

1 a) Area = vertical distance × horizontal distance
$= (8 - 0) \times (10 - 4)$
$= 8 \times 6$
= 48 square units

b) Area = area of triangle
$= \frac{1}{2} \times$ base × height
$= \frac{1}{2} \times 4 \times 8$
= 16 square units

c) Area = area of a rectangle + area of triangle
$= 4 \times 2 + \frac{1}{2} \times 2 \times 4$
$= 8 + 4$
= 12 square units

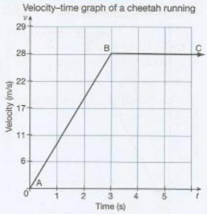
d) Area $= (20 - 12) \times 4$
$= 8 \times 4$
= 32 square units

e) Area $= 48 + 16 + 12 + 32 = 108$ square units

2 The distance the object covered equals the area (108 square units) as the vertical unit (velocity) is in m/s and the horizontal unit (time) is in seconds. When you multiply the values with the units, you will get metres (m/s × s = m). The distance is 108 m.

Activity 7

1 The graph on the right is from Activity 6 question 2 about the motion of a cheetah.



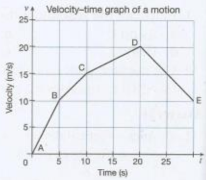
a) Calculate the area under the curve from A to C.

b) Use the area under the curve to give the distance the cheetah covered during the relevant part of the motion.

2 The graph from Activity 6 question 3 is given on the right.

a) Calculate the area under the graph from A to E.

b) Use the area under the graph to give the distance the object covered during the relevant motion.



3 The velocity–time graph on the right shows the motion of an object.

a) Use the graph to find the velocity of the object at $t = 5$ s.

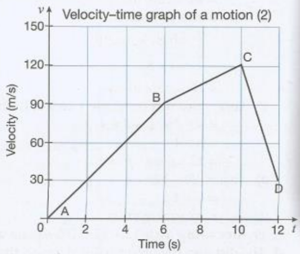
b) After how many seconds did the object have a velocity of 120 m/s?

c) Calculate the acceleration over each time interval.

i) 0 s to 6 s

ii) 6 s to 10 s

iii) 10 s to 12 s



d) Find the distance covered between four seconds and ten seconds.

e) Calculate the total distance covered by the moving object.

Activity 7 (conti

4 The graph show it takes two cars speed of 27.8 m position of rest.

a) Calculate the 27.8 m/s as k per hour (km

b) Car A takes 3 reach a speed Calculate car acceleration.

c) Calculate the car A covers f start until it r speed of 27.8

d) Car B takes 4 a speed of 27. Calculate car

e) Calculate the of 27.8 m/s.

5 Nosiku left home friend. She accel until she reached this constant spe Then she had to ran into the road 25 seconds after

a) Draw a veloci journey.

b) Calculate her home.

c) Calculate the

d) Calculate the

Practical activity

Work in small group people below? Expla

- A long-distance a
- Someone who dr
- Someone in a nat

Activity 7 (continued)

4 The graph shows the time it takes two cars to reach a speed of 27.8 m/s from a position of rest.

a) Calculate the speed of 27.8 m/s as kilometres per hour (km/h).

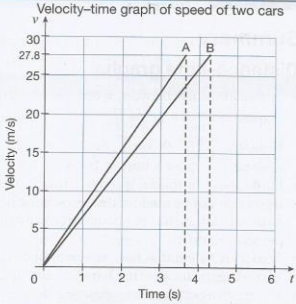
b) Car A takes 3.65 s to reach a speed of 27.8 m/s. Calculate car A's average acceleration.

c) Calculate the distance car A covers from the start until it reaches a speed of 27.8 m/s.

d) Car B takes 4.3 s to reach a speed of 27.8 m/s. Calculate car B's average acceleration.

e) Calculate the distance car B covers from the start until it reaches the speed of 27.8 m/s.

Velocity–time graph of speed of two cars



5 Nosiku left home on her bicycle to visit her friend. She accelerated uniformly for 12 seconds until she reached a speed of 12 m/s. She rode at this constant speed for another 16 seconds. Then she had to brake sharply for a child who ran into the road. Her bicycle came to rest 25 seconds after she braked.

a) Draw a velocity–time graph of Nosiku's journey.

b) Calculate her acceleration when she left home.

c) Calculate the deceleration at which she came to a stop.

d) Calculate the total distance she covered until she came to a stop.

Practical activity

Work in small groups. How could information in this topic be useful for the people below? Explain.

- A long-distance athlete
- Someone who draws up bus timetables
- Someone in a nature conservation who is studying cheetahs

TOPIC 6

Summary, revision and assessment

Summary

Distance–time graphs

- The relationships between speed, distance and time:
  - » speed = $\frac{\text{distance}}{\text{time}}$ or $S = \frac{D}{T}$
  - » time = $\frac{\text{distance}}{\text{speed}}$ or $T = \frac{D}{S}$
  - » distance = speed × time or $D = S \times T$
- In the above formulae, speed is constant (it does not change).
- Units that can be used for distance–time calculations include kilometres (km), hours (h), kilometres per hour (km/h) and metres (m), seconds (s) and metres per second (m/s).
- Scalars are values that have size but not direction.
- Vectors are quantities that have size and direction.
- The gradient of a line on a distance–time graph gives the speed of the object.
- When drawing distance–time graphs or velocity–time graphs (or any graph), always plot the independent variable (time) on the horizontal axis.

Velocity–time graphs

- The gradient of a line on a velocity–time graph represents acceleration.
- Acceleration is given by $\vec{a} = \frac{\text{change in velocity}}{\text{time}} = \frac{\vec{v_f} - \vec{v_i}}{t_f - t_i}$ where $\vec{v_i}$ is the velocity at the start, t_i is the initial time, $\vec{v_f}$ is the final velocity and t_f is the time at the end.
- Acceleration is a vector quantity.
- Negative acceleration means deceleration or retardation (slowing down).
- The area under a velocity–time graph represents the distance covered during the movement.
- We use the concept of similarity to compare the area under a curve with a similar shape that has the correct measurements.

Revision exercises

1 Use the relationship between speed, time and distance to complete a copy of the table.

	Distance	Speed	Time
a)	28 m	...... m/s	4 s
b)	180 km	120 km/h	...... h
c)	..... km	15 km/h	3 h
d)	49 m	2 m/s	..... s
e)	...... km	70 km	4 h

2 Complete a copy
acceleration, velo

	Initial velo
a)	28 m/s
b)	0 m/s
c)	0 m/s
d)	49 m/s

3 A car travels along
for the whole jour
a) Sketch the disp
b) What was the
c) What was the

4 The Comrades Ma
the oldest ultrama
the world. It takes
annually between
Pietermaritzburg
It is run over a dis
The best time for
was set in 2008 by
Leonid Shvetsov.
24 minutes and 47
a) Change the dist
to seconds.
b) Give the averag
c) Convert the wi
d) Write the winn

5 The flying distance
lies at 39° 15' E and
30 minutes.
a) Calculate the d
b) Write down the
to Cairo.
c) Calculate the av
to Cairo.
d) Draw a displace
e) Write down the

2 Complete a copy of the table below using the relationship between acceleration, velocity and time. (Round off answers to one decimal place.)

	Initial velocity	Final velocity	Acceleration	Time
a)	28 m/s	44 m/s	...... m/s	4 s
b)	0 m/s	27.8 m/s	3.5 m/s	...... s
c)	0 m/s	.... m/s	15 m/min.	3 min.
d)	49 m/s	0 m/s	..... m/s	7 s

3 A car travels along a straight road for 600 m. It travels at a constant velocity for the whole journey, which takes 120 s.
   a) Sketch the displacement–time graph for the journey.
   b) What was the velocity of the car in metres per second (m/s)?
   c) What was the velocity of the car in kilometres per hour (km/h)?

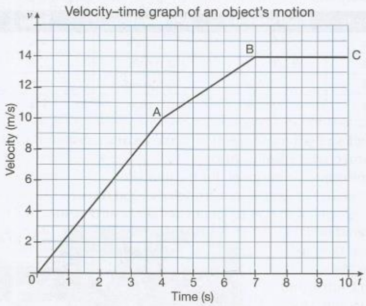
4 The Comrades Marathon is the oldest ultramarathon in the world. It takes place annually between Durban and Pietermaritzburg (South Africa). It is run over a distance of 90 km. The best time for men so far was set in 2008 by the Russian Leonid Shvetsov. It was 5 hours 24 minutes and 47 seconds.

Comrades Marathon runners

   a) Change the distance for the marathon to metres and the winning time to seconds.
   b) Give the average speed of the best time in metres per second (m/s).
   c) Convert the winning time to hours accurate to three decimal places.
   d) Write the winner's speed in kilometres per hour (km/h).

5 The flying distance between Lusaka and Cairo in Egypt is 5 036 km. Cairo lies at 39° 15' E and Lusaka at 28° 15' E. The flying time is 6 hours and 30 minutes.
   a) Calculate the difference in longitude between Lusaka and Cairo.
   b) Write down the displacement of an aeroplane that flies from Lusaka to Cairo.
   c) Calculate the average velocity of an aeroplane that flies from Lusaka to Cairo.
   d) Draw a displacement–time graph to represent the situation.
   e) Write down the displacement of a plane that flies from Cairo to Lusaka.

6 The velocity–time graph shows an object's motion over a time period of 10 s.



a) Read off the velocity of the object at 4 s.
b) After how many seconds was the velocity 12 m/s?
c) Calculate the acceleration from $t = 0$ s to $t = 4$ s.
d) What is the acceleration over the interval 7 s to 10 s?
e) Find the acceleration over the interval 4 s to 7 s.
f) Use the area under the graph to calculate the total distance the object covered in the ten seconds that the motion was observed.

Assessment exercises

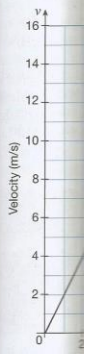
1 a) Find the distance covered at 59 km/h for one and a half hours.
b) Find the time taken to cover 26 km at 52 km/h.
c) Find the speed if 368 km is covered in 8 hours.

2 An object is dropped from a bridge over a river that is 42 m above the water. The initial velocity was 0 m/s and the acceleration is approximately 9.8 m/s^2.
a) How long did it take the object to reach the water if the relationship between the time, the displacement (s) and the acceleration (9.8) is given by the following formula?

$$t = \sqrt{\frac{2 \times 42}{9.8}}\ \text{s}$$

b) Find the velocity of the object as it hits the water using $v = 9.8\,t$.

3 The Comrad
distance of 9
6 hours 9 mi
a) Convert t
b) Convert t
second (m
c) Write the
d) Give the
4 Below is the
traffic lights



a) Calculate
i) 0 s to
ii) 7 s to
iii) 16 s to
b) What was
c) Calculate

3 The Comrades Marathon takes place annually in South Africa. It is run over a distance of 90 km. The best time for women so far was set in 2006. It was 6 hours 9 minutes and 24 seconds.

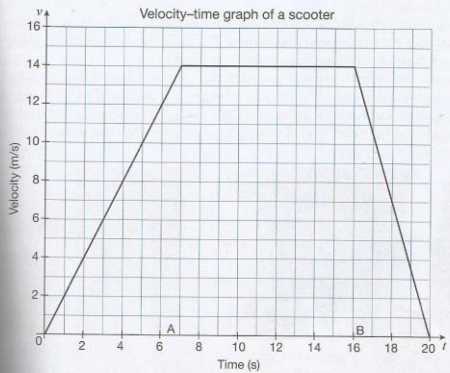
a) Convert the distance to metres and the winning time to seconds.

b) Convert the average speed of the best time for women to metres per second (m/s).

c) Write the winning time in hours accurate to three decimal places.

d) Give the winner's speed in kilometres per hour (km/h).

4 Below is the velocity–time graph for a scooter that moved between two sets of traffic lights on a straight road.



a) Calculate the scooter's acceleration over each time interval.

i) 0 s to 7 s

ii) 7 s to 16 s

iii) 16 s to 20 s

b) What was the velocity of the scooter at 10 s?

c) Calculate the distance between the two traffic lights.

TOPIC 7

Social and commercial arithmetic

Sub-topic	Specific Outcomes
Investments	• Carry out calculations that involve shares, dividends and investment bonds

Starter activity

Work in groups of four.

1 Find out which different investments are offered by banks. Make notes about the advantages and disadvantages of each type of investment. Decide which type of investment is the most profitable.

2 Use the commercial pages of newspapers to find companies that are listed on the stock exchange. Let each member of your group choose one or two companies and track the changes in those companies' share prices over a period of at least one week. Discuss in your group what these changes in the share prices mean.

3 Working with your group, make a list of the places where you can find share prices (two such places are shown in the photographs on this page).

4 What do you think the difference is between a bull market and a bear market?

SUB-TOPIC 1 Ir

Introduction

Every country need and for a society to production of food railways) and educa economy that gene and investment bon

Calculations t

The economy is usu businesses also play on formal businesses issue **shares** in the c into parts (shares), w **exchange** where sha company, they own directors of a compa be given to the share called **dividends**.

The statues of the bu outside the Frankfurt S A bull market describe with rising share price describes a market in general decrease in sh

The number of share determined by the a the share capital int K1 000 000 and it is

SUB-TOPIC 1

Investments

Introduction

Every country needs a healthy, working economy. This is essential for job creation and for a society to work properly. A country needs food security (the steady production of food), good health services, infrastructure (such as roads and railways) and education. People can make sound investments in a healthy economy that generates money. Examples of investments include capital, shares and investment bonds. In this topic, the focus is on shares and investment bonds.

Calculations that involve shares

The economy is usually linked to formal businesses or companies. Informal businesses also play an important role in an economy, but in this topic, the focus is on formal businesses. When a **company** needs cash so that it can expand, it can issue **shares** in the company. This means the company divides some of its capital into parts (shares), which it then offers for sale. The company is listed on the **stock exchange** where shares (also called stock), are traded. When people buy shares in a company, they own a share of the company. They are called **shareholders**. The directors of a company decide what amount of the profit the company makes will be given to the shareholders. This is usually done once a year. These payments are called **dividends**.

The statues of the bull and the bear are outside the Frankfurt Stock Exchange. A bull market describes a strong market with rising share prices. A bear market describes a market in which there is a general decrease in share prices.

New words

company: a structured commercial business or corporation
dividends: the part of a company's profit that is paid to shareholders on a regular basis
shares: parts of the capital a company offers for sale in the form of shares
shareholders: the people who have bought shares of a company
stock exchange: the financial market of a country where shares are traded

Did you know?

The Lusaka Stock Exchange (LuSE) is the main stock exchange in Zambia. It was founded in 1993. It is situated in Lusaka.

The number of shares a company can make available to shareholders is determined by the amount of share capital the company has and the division of the share capital into shares of a fixed amount. For example, if a company has K1 000 000 and it issues 4 000 000 shares, the nominal value of one share will be

K0.25 (usually given as 25 Ngwee). This nominal price has nothing to do with the market value of the shares. The market value is the price prospective shareholders pay for the shares or the value at which shareholders can sell shares.


Kwacha notes

Worked example 1

1 Mr Chona bought 500 shares of a company at K2 500 per share. The nominal value of a share was K1 000.
   a) What did he pay for the 500 shares?
   b) What is the total nominal value of the shares?

2 The table below shows part of the Lusaka Stock Exchange (LuSE) listing for one day. Answer the questions.

1	2	3	4	5	6	7
Company name	Exch	Curr	Close	Change	Volume	Value
Zambia National Commercial Bank Plc (Zanaco)	LuSE	ZMK	0.28	+0.00	95 494	

   a) What do the last six columns show?
   b) How many shares were traded on that day?
   c) What was the value of the shares that were traded on that day?

Answers

1 a) Cost of shares: 500 × K2 500 = K1 250 000
   b) Total nominal value: 500 × K1 000 = K500 000

2 a) Column 2 shows the stock exchange on which the company is noted.
   Column 3 shows the currency in which the company traded.
   Column 4 shows the closing price for one share.
   Column 5 shows the change in the share price from the previous day.
   Column 6 shows the number of shares that were trade on that day.
   Column 7 shows the total value of the shares that were traded on that day.
   b) 95 494 shares
   c) Value = closing price × volume = 0.28 × 95 494 = 26 738.32 Ngwee

A person who buys and sells goods and assets for others is called a broker. A **stockbroker** is a regulated professional broker who buys and sells shares and bonds. Brokers earn money by charging a **brokerage fee** or commission on the shares and bonds they buy and sell for others.

New words

stockbroker: person who buys and sells shares and bonds for others
brokerage fee: *the fee charged by a stockbroker for buying and selling* shares and bonds

Activity 1

1 Mrs Chimbala
Ltd shares at a
bought the sh
brokerage fee
   a) Calculate th
   b) What did s

2 The table belo
a particular da
rounded off to

Company name
First Quantum Minerals Ltd
First Quantum Minerals Ltd (ZDR)
Lafarge cement Zambia Plc
Metal Fabricators o Zambia Plc
Zambian Breweries
Zambeef Products

3 What was the
   a) Lafarge cem
   c) Zambeef Pr

4 Malika buys 20
so do its profits
pay K140 for a
   a) What did M
   b) Malika deci
   she gets for
   c) Calculate th
   d) Calculate he

5 Isake has a com
so he issues 10
   a) Give a reas
   b) How many
   c) How much
   d) After four y
   percentage
   e) What is the

Activity 1

1 Mrs Chimbala bought 5 500 First Quantum Minerals Ltd shares at a price of US$18.52. A stockbroker bought the shares for her and she had to pay a brokerage fee of 1.5% of the total purchase price.
   a) Calculate the purchase price of the shares.
   b) What did she pay the stockbroker?


US dollar bills

2 The table below shows part of the LuSE listing for a particular day. Calculate the values of (a) to (f) rounded off to two decimal places.

Company name	Exch	Curr	Closing price	Change	Volume	Value
First Quantum Minerals Ltd	OTCBB	USD	18.99	+0.45	24 900	**a)**
First Quantum Minerals Ltd (ZDR)	LuSE	ZMK	4.00	+0.00	**b)**	4 000.00
Lafarge cement Zambia Plc	LuSE	ZMK	**c)**	-0.25	4 333	37 827.09
Metal Fabricators of Zambia Plc	LuSE	ZMK	0.47	+0.00	589 000	**d)**
Zambian Breweries Plc	LuSE	ZMK	3.32	–0.01	**e)**	2 476.72
Zambeef Products Plc	LuSE	ZMK	4.20	+0.50	1 343 692	**f)**

3 What was the price of the following on the previous day?
   a) Lafarge cement Zambia Plc
   b) Zambian Breweries Plc
   c) Zambeef Products Plc
   d) Metal Fabrications of Zambia Plc

4 Malika buys 200 shares in a company at K100 each. The company grows and so do its profits. The demand for the shares rises and people are prepared to pay K140 for a share.
   a) What did Malika pay for her 200 shares?
   b) Malika decides to sell her shares at K140 each. Calculate the amount that she gets for her shares.
   c) Calculate the profit (or capital gain) that Malika made on her shares.
   d) Calculate her profit as a percentage.

5 Isake has a company. He wants to sell 45% of the shares in his company and so he issues 100 000 shares. He keeps the rest of the shares.
   a) Give a reason why Isake probably wants to sell shares in his company.
   b) How many shares will Isake make available for shareholders to buy?
   c) How much money will he raise if the price per share is K23.00?
   d) After four years, the value of the shares has risen to K56.00 each. By what percentage has the share price increased?
   e) What is the stock of Isake's company worth after four years?

Calculations that involve dividends

People who invest in shares (or stocks) hope to share in the profit of a company. This happens when:

- they sell shares at a profit
- a dividend is paid out (for example, once a year) while they own the shares.

The directors of a company decide what portion of the annual profit will be distributed to shareholders. The dividend per share is then calculated as follows:

$$\text{Dividend per share} = \frac{\text{total dividend amount}}{\text{number of shares issued}}$$

Worked example 2

Isake owns 55% of the shares in his company. The company made a profit of K2 560 000. An amount of K1 400 000 was paid out as dividends to the shareholders who had bought 100 000 shares.

1 Calculate the amount paid out as dividend on one share.
2 What amount was paid out in dividends to the shareholders?

Answers

1 $\text{Dividend per share} = \frac{\text{total dividend amount}}{\text{number of shares issued}} = \frac{\text{K1 400 000}}{\text{100 000}} = \text{K14}$

2 Amount paid = 45% of 100 000 × K14 = 45 000 × K14 = K630 000

Activity 2

1 The directors of a company decide to pay a total dividend of K978 000 on 1 250 000 shares.
  a) Calculate the dividend per share.
  b) Leya holds 15 000 shares in the company. How much is paid out in dividends to her?
2 Calculate the dividend per share in each case.
  a) A total dividend of K2 135 000 is paid on 2 000 000 shares.
  b) A total dividend of K2 850 000 is paid on 250 000 shares.
3 An investor uses the following formula to calculate his annual dividend yield.
  $$\text{Annual dividend yield} = \frac{\text{dividend paid}}{\text{total cost of shares}} \times 100$$
  Tina bought 2 500 shares at K5.85 per share. She received a dividend of K0.59 per share after a few months.
  a) Calculate the total amount she paid for her shares.
  b) Calculate the dividend she received on her shares.
  c) Show that the dividend yield is 10.9%.
  d) Tina's stockbroker charges a brokerage fee of 1.4% on transactions. Calculate the amount she had to pay the stockbroker.

Calculations

An investment b
money in the bor
government or a
bank can supply.
by taking loans in
investors. They m
interest. These in
stadium, a road, a
governments are
The length of t
someone lends m
amount a bond p
higher yield in in
people who make
to them for a long
matures is the dat
of investment is a
advance what the
Bonds are usual
example, you cou
K1 000. The inter
coupon when the
the bond (the face
to the certificate.

Worked exam

Milimo buys a b
after 10 years. Th
1 Calculate th
2 How much i
3 What amou
4 How much i
rate remains

Answers

1 Interest: 8%
2 After six mo
3 He receives
six months.
4 Total interes

Calculations that involve investment bonds

An investment bond is like a loan, where you are the lender. You invest (loan) money in the borrower in the form of bonds. The borrower is usually a government or a municipality that needs larger sums of money than the average bank can supply. The municipality then turns to the public and raises the money by taking loans in smaller amounts in the form of bonds issues, from thousands of investors. They make a promise to pay you back in full with regular payments of interest. These institutions may sell these bonds to raise money for building a stadium, a road, a bridge or to finance increasing debts. Debts issued by governments are often called *treasuries*.

The length of time for which someone holds a bond (the length of time someone lends money to the issuer of the bond) also influences the yield (the amount a bond produces). For example, a bond that is held for 10 years will pay a higher yield in interest than a bond that is issued for one year. In effect, the people who make such a loan are being paid for not having their money available to them for a longer period of time. The date on which a bond or investment matures is the date by which the lender has promised to repay the bond. This type of investment is also called a fixed income investment (as someone knows in advance what the amount is that they will receive on maturity of the bond).

Bonds are usually available in set units (for example, in units of K100). For example, you could buy 10 units at K100 and then your investment would be K1 000. The interest rate is sometimes called a *coupon*. The investment was called a coupon when the system was started. A certificate was issued for the amount of the bond (the face value) and a coupon that showed the interest rate was attached to the certificate.

Worked example 3

Milimo buys a bond worth K100 000 that pays interest of 8% **p.a.** It matures after 10 years. The interest is paid to him every six months.

1 Calculate the interest payable per year.
2 How much interest will Milimo receive after six months?
3 What amount will he receive on maturity of the bond?
4 How much interest will he receive in total if the interest rate remains the same?

New word

p.a.: per annum (per year)

Answers

1 Interest: 8% of K100 000 = K8 000
2 After six months he receives K4 000 in interest.
3 He receives only the original amount (K100 000) as interest was paid every six months.
4 Total interest: 10 × K8 000 = K80 000

Activity 3

1 A municipality needs K15 000 000 to finance the building of a football stadium. They sell bonds payable in five year's time at an interest rate of 6% p.a.
   a) How many units do they need to sell if a unit sells for K500?
   b) How much interest will they pay in one year?
   c) Calculate the total amount of interest paid after five years.
   d) Mwila bought 10 bond units as an investment. What amount did Mwila invest?
   e) What amount will Mwila get as interest over the five years?

2 Katele takes up an investment bond with a bank. The interest rate is 5% and the interest amount per year (which is paid in two payments) is K1 000.
   a) How much interest does Katele receive every six months?
   b) What was the value of Katele's bond with the bank?

3 Nabila takes a government bond worth K10 500 for 15 years. The interest rate is 6%. Calculate the interest she earns in one year.

4 Nabila decided to deposit the interest she received as calculated in question 3 in a savings account at a rate of 7.5% compound interest per annum. She managed to continue this for four years.

> **Remember**
>
> **compound interest:** interest is not used but added to the capital for the next compounding period

   a) Calculate the growth of the first amount Nabila deposited after four years.
   b) Calculate the amount for the second year's deposit after three years.
   c) Calculate the amount for the third year's deposit after two years.
   d) By how much did the last amount she deposited grow in the fourth year?
   e) Find the amount in her savings account after four years (to the nearest kwacha).

5 A company issued a bond of K2 750 000 at a rate of 4.5% to mature after 12 years.
   a) What is the interest paid over the full period?
   b) If the bond is paid back after seven years, what is the full amount of the face value and the interest to that date?

6 In order to start an **aviation** company, a government needed K3 billion but the Commercial Bank could only award a bond of K2 billion at an interest rate of 12% p.a. If the Bank of Zambia was ready to pay the deficit at 4%, how much would the government have had to pay to the Commercial Bank and the Bank of Zambia in total?

> **New word**
>
> **aviation:** the activity or business of operating and flying aircraft

TOPIC 7

Summary

Shares

- Companies often
- Companies offer shares.
- Companies are
- Shareholders are share of the

Dividends

- A part of the shareholders. called dividends.
  - Dividend per

Bonds

- An investment is the lender.
- The borrower is needs a lot of
- The borrower from thousands
- The borrower regular payments

1 Calculate the
   a) 900 Lafarge is charged
   b) 1 250 Copper
   c) 4 500 ZCCM

2 Liya bought 1 per share.
   a) Calculate the
   b) How much
   c) Liya decided she make?

TOPIC 7 Summary, revision and assessment

Summary

Shares

- Companies often issue shares in the company when they need to raise cash.
- Companies offer some of the growth in the company for sale in the form of shares.
- Companies are listed on the stock exchange where shares (or stock) are traded.
- Shareholders are people who have bought shares in a company; they own a share of the company.

Dividends

- A part of the profit a company makes is divided between the shares and paid to shareholders. This is usually done once a year. The money that is paid out is called dividends.
  - » Dividend per share = $\frac{\text{total dividend amount}}{\text{number of shares issued}}$

Bonds

- An investment bond is similar to a loan, where the person who holds the bond is the lender.
- The borrower is usually a government, a municipality or an institution that needs a lot of money.
- The borrower raises money by making small loans in the form of bonds issues from thousands of investors.
- The borrower makes a promise to pay shareholders back in full by making regular payments that include interest.

Revision exercises

1. Calculate the cost of buying the following.
   a) 900 Lafarge Cement Zambia Plc shares at K8.90 each; a brokerage fee of 2% is charged
   b) 1 250 Copperbelt Energy Corporation Plc shares at K0.84 each
   c) 4 500 ZCCM Investment Holdings Plc shares at €2.00 (euros) per share
2. Liya bought 1 675 shares at K5.20. She received an annual dividend of K0.29 per share.
   a) Calculate the total amount she paid for the shares.
   b) How much was her annual dividend?
   c) Liya decided to sell her shares at K8.59 per share. How much profit did she make?

Revision and assessment (continued)

3 At the beginning of June 2012, the turnover of a company increased by 12.5% to K153 000 000. The dividend rose from K0.14 to K0.19 per share.
   a) Calculate the company's turnover before the increase in turnover.
   b) Calculate the percentage increase in the dividend per share.

4 In 2009, a company issued 250 000 bonds at K25 each and 100 000 bonds at K50 each for maturity after 15 years at 5.5% interest p.a.
   a) How much cash could the company generate if it sold all the bonds?
   b) How much interest would the company pay if all the bonds were sold when they were issued and all the bonds were repaid on the maturity date?

5 Suzyo bought 25 bonds at K25 each when the company mentioned in question 4 issued bonds in 2009.
   a) How much interest did she receive on her 25 bonds in 2009?
   b) In 2010, Suzyo bought another 15 bonds at K50 each. How much interest did she receive on these 15 shares annually?
   c) What was the total amount of the interest Suzyo received on her bond investment after 15 years?

Assessment exercises

Njungu bought 7 500 shares for K12.50 each at a brokerage fee of 1.7% from a company. After a year, the company paid a dividend of K0.63 per share.

1 a) Calculate the cost of the shares without the brokerage fee.
   b) What was the amount of the brokerage fee?
   c) Calculate the amount the company paid Njungu as an annual dividend.

2 The value of the shares that Njungu bought increased. When the market value per share was K28.23, he sold his 7 500 shares.
   a) If the dividend per share was K1.21 when Njungu sold his shares, what was the total dividend he received when he sold all his shares?
   b) What would Njungu have received for his shares if he had sold them at K23.23 per share?
   c) How much profit did he make when he sold his shares at K28.23 each?

3 Njungu decided to invest his profit in investment bonds that were issued by the municipality at K150 each.
   a) How many bonds could Njungu buy with the profit he made from the sale of his shares?
   b) If the interest rate paid on the investment bonds was 6%, what amount of interest did Njungu receive as interest anually?
   c) The period to maturity of the bonds was seven years. How much money did Njungu receive in total when the bonds matured? Include interest in the total.

TOPIC 8

B

Sub-topic

Bearings and scale drawings

Starter activity

Work with a partner.
Refer to the map bel
relation to Kitwe and

TOPIC 8

Bearings

Sub-topic	Specific Outcomes
Bearings and scale drawings	• Draw/sketch diagrams to represent position and direction. • Use bearing and scale drawing in real life.

Starter activity

Work with a partner.
Refer to the map below. Discuss how to explain the direction of Solwezi in relation to Kitwe and Kasempa.

SUB-TOPIC 1 Bearings and scale drawings

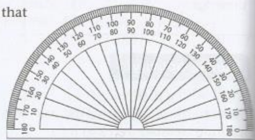
Bearing refers to the direction of a movement. We use three methods to show direction:

- compass bearings (such as N 42° E and S 53° E)
- coordinates that can be used by a GPS (such as 15.3306° S, 28.4525° E for Lusaka International Airport)
- bearings that describe direction in terms of an angle measured from north.

In this topic, you will learn about bearings that are given as angles measured from north.

Draw and measure angles

In order to learn about bearings, you must be able to draw and measure angles accurately.

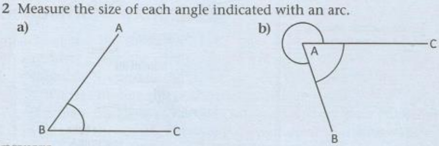


Protractor

Worked example 1

1 Draw an angle of 63°.

2 Measure the size of each angle indicated with an arc.



Answers

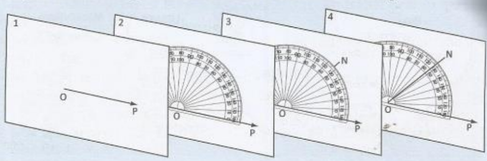
1 Refer to the series of drawings below.

Step 1 Draw a line OP.

Step 2 Place a protractor on the line OP with its centre at O.

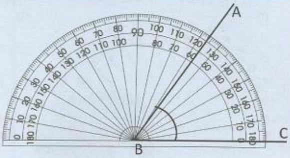
Step 3 Make a mark N on the paper next to 63° on the protractor.

Step 4 Remove the protractor and draw a line from O through N.



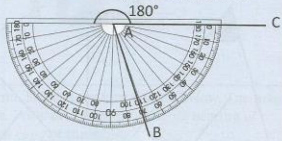
Worked example 1 (continued)

2 a) Place the centre of the protractor at B and read off the angle on the inner scale of the protractor.
$C\hat{B}A = 56°$



b) The angle is greater than 270°. There are two ways of finding its size:

- Place the centre of the protractor at A that side AC lies on the edge of the protractor and side AB is covered by the protractor. Read off the angle on the inner scale. It is 109°. Add 180° to this reading to find the size of the angle. It is 289°.



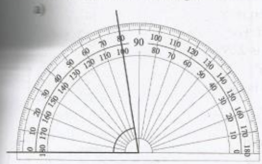
- Place the protractor as above, but start on side AC and read off the size of the acute angle CAB. It is 71° on the outer scale of the protractor. Subtract this value from 360° to get the required size of the angle, 289°.

Activity 1

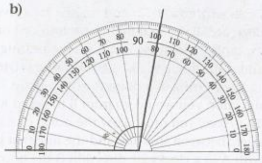
1 Draw each angle.
   a) 52°
   b) 120°
   c) 222°
   d) 351°

2 Give the size of each angle.

a)

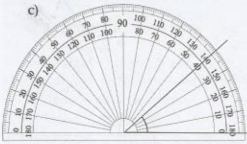


b)

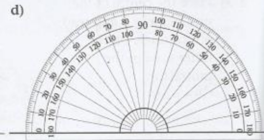


Activity 1 (continued)

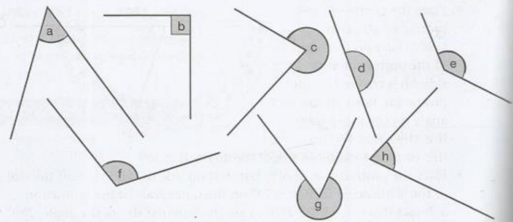
c)



d)



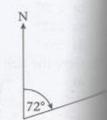
3 Measure the size of angles a to h.



Bearings and scale drawings

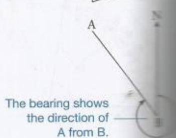
You can give a bearing as an angle on its own or you can combine a bearing with a distance.

Three-figure bearings



A bearing is an angle that is measured clockwise from north. It is given as three figures, for example 72° is written as 072° and we say: zero seven two degrees. All decimal figures are rounded off. For example, if an angle is 224.6°, the bearing is 225° and if the angle is 46.2°, the bearing is 046°.

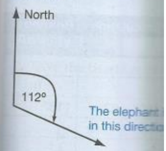
The bearing of a point A from a point B is the direction you would go from B to get to A. Therefore, you must draw the north line at B. The angle between this line and the line that joins B with A in a clockwise direction is the bearing.



Worked example 2

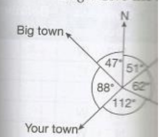
Make a drawing to illus
a bearing of 112°.

Answer

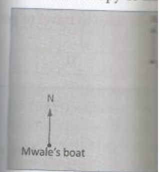


Activity 2

1 Katembe is standing o
shows the position of
each village. Give the



2 At Lake Bangweulu, the
fish. Make a copy of th

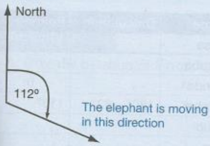


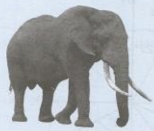
The bearing of the shoa
the fish from Mwale's bo
a) Draw the bearings fo
b) Mark the position of

Worked example 2

Make a drawing to illustrate the direction of an elephant that is walking at a bearing of 112°.

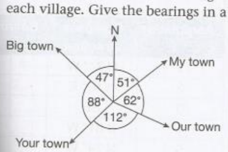
Answer





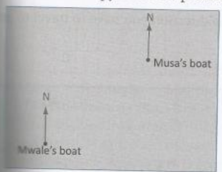
Activity 2

1 Katembe is standing on top of a hill. She can see four villages below. The map shows the position of each village. Write down the three-figure bearing of each village. Give the bearings in a copy of the table.



Village	Bearing
My town	
Our town	
Big town	
Your town	

2 At Lake Bangweulu, the fishermen in two boats have spotted a large shoal of fish. Make a copy of the map and then complete the instructions.



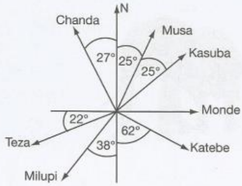
The bearing of the shoal of fish from Musa's boat is 255° and the bearing of the fish from Mwale's boat is 036°.

a) Draw the bearings for both boats on your map.

b) Mark the position of the shoal of fish on the map.

Activity 2 (continued)

3 A taxi pulls up at a taxi rank and eight people got out. The drawing shows the direction in which each person walked. Work out each person's bearing. Fill your answers in on a copy of the table. (Katebe's bearing has been completed as an example.)



Name	Calculation	Bearing
Musa		
Kasuba		
Monde		
Katebe	180° – 62°	118°
Milupi		
Teza		
Chanda		

4 Five aeroplanes take off from an airport and fly according to the compass directions as shown in the table. Calculate each aeroplane's three-figure bearing and fill it in on a copy of the table.

Aeroplane	Compass direction	Bearing
1	N 34° W	
2	N 52° E	
3	SE	
4	S 81° W	
5	N 74° W	

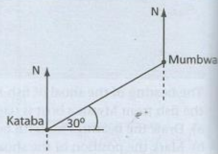
Bearings of objects relative to one another

The bearing of a point A from a point B is the direction you have to travel to get from A to B.

Worked example 3

The drawing shows the relative positions of Mumbwa in Central Province and Kataba in Western Province.

1 Find the bearing of Kataba from Mumbwa.
2 Find the bearing of Mumbwa from Kataba.



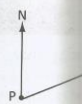
Worked exam

Answers

1 Clockwise a
2 Clockwise a

Activity 3

1 Give the bear
a)



2 Make a copy o
your own grid

a) Measure the
b) Find the be
c) Find the be
3 Draw points M
draw the points
a) A is on a be
b) B is on a be
c) C is on a be
d) D is on a be

Worked example 3 (continued)

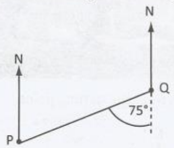
Answers

1 Clockwise angle from the north line at Mumbwa: 240°
2 Clockwise angle from the north line at Kataba: 060°

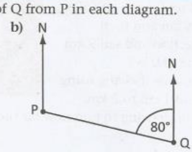
Activity 3

1 Give the bearings of P from Q, and of Q from P in each diagram.

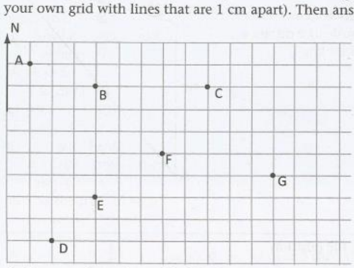
a)



b)



2 Make a copy of the points on the grid below on 1 cm graph paper (or draw your own grid with lines that are 1 cm apart). Then answer the questions.



a) Measure the angles to find the bearings from A of B, C, D, E, F and G.
b) Find the bearings from B of A, C, D, E, F and G.
c) Find the bearings from D of E, F, C and G.

3 Draw points M and N on the same horizontal line on a clean page. Then draw the points A, B, C and D according to the instructions below.
a) A is on a bearing of 051° from M and on a bearing 020° from N.
b) B is on a bearing of 097° from M and on a bearing 125° from N.
c) C is on a bearing of 119° from M and on a bearing 135° from N.
d) D is on a bearing of 322° from M and on a bearing 295° from N.

Bearings and scales

When we combine bearings and distance, we usually need to make a scale drawing. The bearing is given by the angle and the distance according to a scale, for example 1 cm equals 1 km.

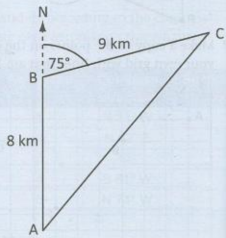
Worked example 4

Two fishermen leave their village and row north on Lake Kariba for 8 km and then change direction and sail 9 km on a bearing of 075°.

1 Make a scale drawing using a scale of 1 cm to 2 km.
2 Use your drawing to find how far the boat is from the starting point.

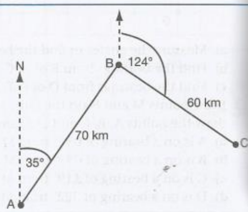
Answers

1 The diagram is drawn to a scale 1 cm = 2 km.
2 AC is approximately 6.7 cm. According to the scale, the distance is approximately 13.4 km.



Activity 4

1 The drawing shows the position of three towns.
   a) Make a scale drawing that shows the positions of the three towns (A, B and C).
   b) Give the bearing of B from C.
   c) Find the bearing of C from A.
   d) Use your drawing to find the approximate distance from A to C.



Activity 4 (con

2 A boat leaves i
a bearing of 15
and sails for 10
Make a scale d
scale 1 cm = 2
a) How far is t
when it gets
b) What shoul
it returns to

3 The drawing o
grid shows the
of three campi
1 cm on the gri
represents 1 km
a) How far nor
A is camp B?
b) How far wes
C is camp A?
c) Measure the
camp B from
d) Measure the
between cam
camp C.
e) Use the scale
the actual di
between cam

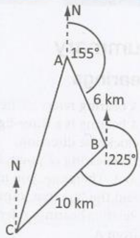
4 A herd of buffal
a bearing of 270
a) Make a roug
b) Make a scale
the situation
c) Measure the
animals are f
which they s
d) Use calculati
answer.

Activity 4 (continued)

2 A boat leaves its mooring and sails 6 km on a bearing of 155° and then it changes direction and sails for 10 km on a bearing of 225°.
Make a scale drawing of the situation, using the scale 1 cm = 2 km.

a) How far is the boat from its starting point when it gets to point C?

b) What should the bearing of the boat be when it returns to point A?



3 The drawing on the grid shows the position of three camping sites. 1 cm on the grid represents 1 km.

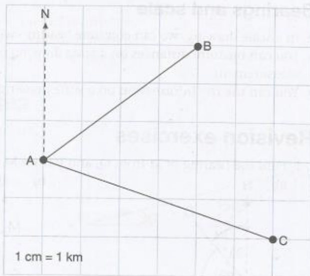
a) How far north of camp A is camp B?

b) How far west of camp C is camp A?

c) Measure the bearing of camp B from camp A.

d) Measure the distance between camp A and camp C.

e) Use the scale to write the actual distance between camp A and camp C.



4 A herd of buffalo travel due south for 6 km and then changes direction to a bearing of 270° and then continues for another 8 km.

a) Make a rough drawing to represent the situation.

b) Make a scale drawing of the situation.

c) Measure the distance the animals are from the point at which they started walking.

d) Use calculations to check your answer.

TOPIC 8 Summary, revision and assessment

Summary

Bearings

- A bearing refers to the direction of movement.
- A bearing is a three-figure angle such as 036° measured from north in a clockwise direction.
- The bearing of point A from point B is the direction you would go from B to get to A. Therefore, you must draw the north line at B. The angle between this line and the line that joins B with A in a clockwise direction is the bearing.
- Relative bearings refer to the bearing of A from B, and then the bearing of B from A.

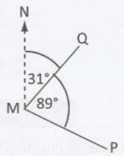
Bearings and scale

- In a scale drawing, we can combine bearings with distance.
- You can measure distances on a scale drawing using the scale to the measurement.
- You can use the information on a scale drawing to calculate distances.

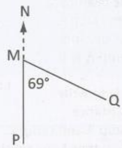
Revision exercises

1 Find the bearing of M from Q, and P from M on each diagram.

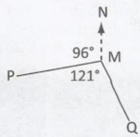
a)



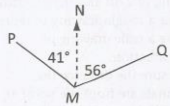
b)



c)



d)



2 Write down each di
a) N 30° E
b) East
c) S 34° E
d) W
e) NW
f) S 55° W

3 The bearing of M fr

4 The bearing of M fr

5 Draw the points A a
of paper. Mark the p
information given b
a) M is on a bearing
b) Q is on a bearing
c) O is on a bearing
d) P is on a bearing

Assessment ex

1 Find the following o
a) the bearing of M f
b) the bearing of Q f
c) the bearing of Q f
d) the bearing of M f
e) the bearing of P f
f) the bearing of P f

2 The bearing of P from
What is the bearing o

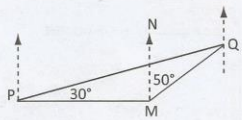
3 The bearing of P from

4 Draw the points A an
of paper. Mark the po
information given be
a) P is on a bearing
b) Q is on a bearing
c) R is on a bearing
d) S is on a bearing

2 Write down each direction as a three-figure bearing.
   a) N 30° E
   b) East
   c) S 34° E
   d) W
   e) NW
   f) S 55° W
3 The bearing of M from Q is 205°. What is the bearing of Q from M?
4 The bearing of M from Q is 052°. What is the bearing of Q from M?
5 Draw the points A and B, the one vertically above the other on a clean piece of paper. Mark the points M, Q, O and P on the paper according to the information given below.
   a) M is on a bearing 095° from A and 023° from B.
   b) Q is on a bearing 265° from A and 318° from B.
   c) O is on a bearing 136° from A and 098° from B.
   d) P is on a bearing 195° from A and 223° from B.

Assessment exercises

1 Find the following on the diagram.
   a) the bearing of M from Q
   b) the bearing of Q from P
   c) the bearing of Q from M
   d) the bearing of M from P
   e) the bearing of P from M
   f) the bearing of P from Q



2 The bearing of P from Q is 336°. What is the bearing of Q from P?
3 The bearing of P from Q is 125°. What is the bearing of Q from P?
4 Draw the points A and B, the one vertically above the other on a clean piece of paper. Mark the points P, Q, R and S on the paper according to the information given below.
   a) P is on a bearing 084° from A and 015° from B.
   b) Q is on a bearing 289° from A and 322° from B.
   c) R is on a bearing 141° from A and 088° from B.
   d) S is on a bearing 205° from A and 343° from B.

5 Use the map below. Find the bearing of:
   a) Lusaka from Zambezi
   b) Zambezi from Solwezi
   c) Kasama from Lundazi
   d) Ndola from Katete
   e) Katete from Ndola
   f) Lundazi from Kasama.

TOPIC
9

Sub-top

The symmetry of

Starter activit

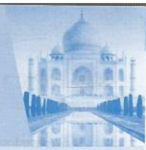
Work with a par
1. Use the pict

2. List five sha

TOPIC

9

Symmetry



Sub-topic	Specific Outcomes
The symmetry of solids	• Determine the order of rotational symmetry • Determine the symmetry of solids • Determine plane symmetry

Starter activity

Work with a partner.

1. Use the picture and the photograph to explain what is meant by symmetry.

2. List five shapes or objects that show symmetry.

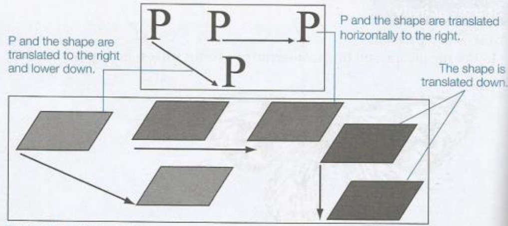
SUB-TOPIC 1 The symmetry of solids

Before we look at the symmetry of solids, we need to make sure we know what is meant by the symmetry of a flat shape (two-dimensional).

Types of symmetry

There are four types of symmetry: translation, reflection, glide reflection and rotation.

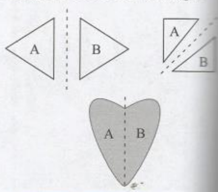
- **Translation:** To translate a shape or an object, move it in a straight line without rotating or reflecting it, or changing it in any other way. We describe a translation by giving the direction and distance a shape or an object is moved.



Examples of translation

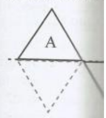
- **Reflection:** To reflect a shape or object about a line is like placing a mirror on the line and seeing the shape reflected in the mirror. The line about which a shape or an object is reflected is called a line of symmetry. You can make a reflection by folding a piece of paper, tracing a shape onto the fold, cutting it out and unfolding the paper.

  Triangle A is reflected in the mirror line (shown by the dotted line) to give the image, triangle B. You can fold the heart along the dotted line so that the two halves fit exacly on one another.



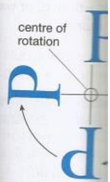
Examples of reflection

- **Glide reflectio**
translation alo
in more than o



Example of a gli

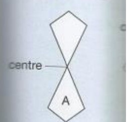
- **Rotation:** To ro
fixed point. The
The letter P has
Each position af



Example of a rota

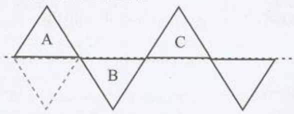
Worked exampl

Look at the diagra



1 a) Through wh
onto the ori
b) How many t
original sha

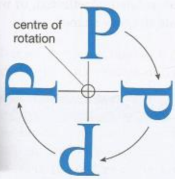
- **Glide reflection:** To create a glide reflection, we combine a reflection with a translation along the mirror line. This is the only type of symmetry that is done in more than one step.



The diagram shows how triangle A was copied, reflected and then translated to give triangle B. Triangle C is a translated copy of triangle A.

Example of a glide reflection

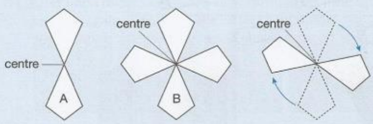
- **Rotation:** To rotate a shape or an object, we turn it through an angle about a fixed point. The point is called the centre of the rotation.
The letter P has been rotated in a clockwise direction through 90° three times. Each position after a rotation is called an image of the original shape.



Example of a rotation

Worked example 1

Look at the diagrams below.



Use the last diagram to help you answer questions 1 and 2.

1 a) Through what angle should shape A be rotated so that the image will fit onto the original shape again?
  b) How many times during a full rotation will the image fit onto the original shape A?

Worked example 1 (continued)

2 a) Through what angle should shape B be rotated so that the image fits onto the original shape again?

b) How many times during a full rotation will the image fit onto the original shape B?

Answers

1 a) 180° clockwise or anticlockwise

b) once

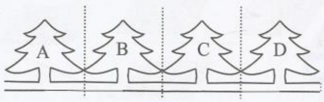
2 a) 90° clockwise or anticlockwise

b) four times

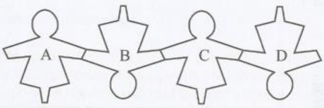
Activity 1

Look at the drawings below. In each case, describe what happened to shape A to create the other shapes. For example, was A translated, rotated or reflected, or was a combination of more than one process used to create the new shape?

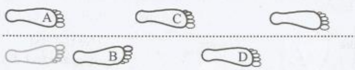
1



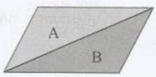
2



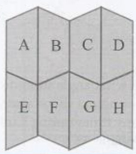
3



4



5



Determine the order of rotational symmetry

A shape or an object has rotational symmetry if the rotated image looks like the original shape or object. The order of rotational symmetry is the number of times the image **coincides** with the original shape or object within a full turn of 360°. For example:

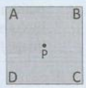
> **New word**
>
> **coincide:** identical, synchronise, correspond, match

- Shape A has rotational symmetry of order 2 (or two-fold symmetry). The image fits onto the original shape twice in one full rotation. The rotational symmetry of order 2 is also called point symmetry.
- Shape B has rotational symmetry of order 4 (or four-fold symmetry).
- The letter P has rotational symmetry of order 1.

Notice that you could also reflect shape A about a horizontal line through the centre to achieve the same symmetry.

Worked example 2

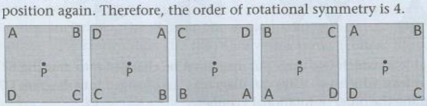
The labels A, B, C and D are not part of the square on the right. Each one refers to one vertex of the square.



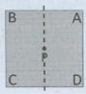
1. Find the order of rotation of a square if its centre of rotation is the point P where the diagonals would intersect.
2. Draw a copy of the square as it would look after a reflection in a line drawn through P, parallel to BC.

Answers

1. After four consecutive turns, each through 90°, the square is in its original position again. Therefore, the order of rotational symmetry is 4.

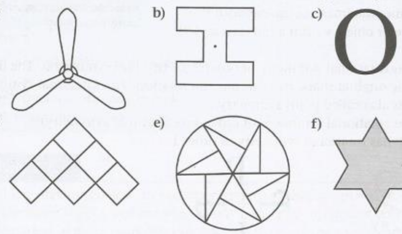


2. The positions of A and B are interchanged, and so are the positions of C and D.



Activity 2

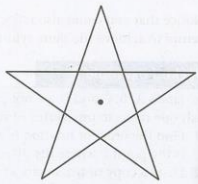
1 Which shapes below have reflection symmetry, which have rotational symmetry, and which have both types of symmetry? Give the order of rotational symmetry for shapes that have rotational symmetry.



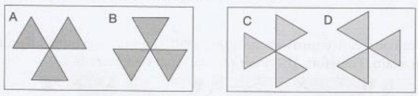
2 Make a copy of the diagram on tracing paper or paper you can see through.

a) Draw all the lines of symmetry on the copy of the drawing.

b) Rotate the copy around its centre point and find the order of rotation by counting the number of times the image falls on the original shape.



3 Look at the two sets of diagrams.

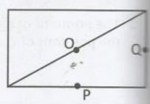


a) In at least which two ways can diagram A be changed into drawing B?

b) In at least which two ways can diagram C be changed into diagram D?

4 Determine the order of rotational symmetry through which the rectangle has been rotated through the three points.

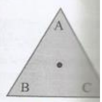
a) O   b) P   c) Q



Activity 2 (con

5 Does the recta
describe each

6 Each shape be
same length a



Triangle

a) Find the ord
through the

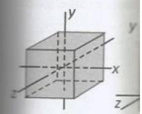
b) Make a copy
shape is refl

c) Complete a

Triangle
Pentagon
Hexagon

Determine the

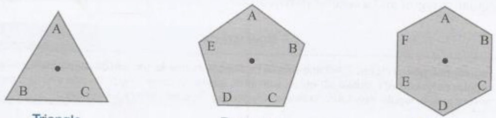
A solid is a 3D-objec
When we draw a
usual x-axis and y-a
the z-axis, is perpen
(see the diagram be
centre of the cube a
the origin.



Activity 2 (continued)

5 Does the rectangle in question 4 have a line (or lines) of symmetry? If so, describe each one.

6 Each shape below is a regular polygon. This means that all its sides are the same length and all its angles are equal.



Triangle Pentagon Hexagon

a) Find the order of rotational symmetry for each shape if it is rotated through the point at its centre.

b) Make a copy of each shape and draw its lines of symmetry. (Remember, a shape is reflected about its line of symmetry.)

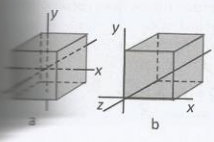
c) Complete a copy of the table for the three shapes.

	Order of rotational symmetry	Number of lines of symmetry
Triangle		
Pentagon		
Hexagon		

Determine the symmetry of solids

A solid is a 3D-object such as a cube, a pyramid or a sphere.

When we draw an object on a system of axes, we should have three axes. The usual x-axis and y-axis are perpendicular to each other in a plane. The third axis, the z-axis, is perpendicular to the plane that is formed by the x-axis and the y-axis (see the diagram below). The diagram shows two positions of a cube, one with the centre of the cube at the origin and the other with a vertex (corner) of the cube at the origin.



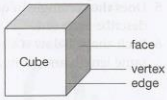
Remember

2D: two-dimensional (shape)
3D: three-dimensional (object)

Rotational symmetry

If a 2D-shape has rotational symmetry, it will have a centre of rotation, but a 3D-object will have an axis of rotation.

We will compare the rotational symmetry of a regular polygon and a regular polyhedron.

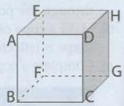


New words

regular polygon: a closed 2D-shape with all its sides the same length and all angles the same size
regular polyhedron: a closed 3D-object with all its edges the same length; therefore all its faces are congruent, regular polygons; there are only five regular solids (the platonic solids)

Worked example 3

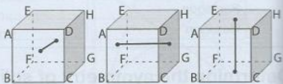
You already know that a square has rotational symmetry of order 4. Using the cube on the right, find the number of axes of symmetry of a cube.



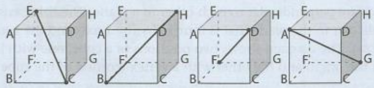
Answer

There are 13 axes of symmetry:

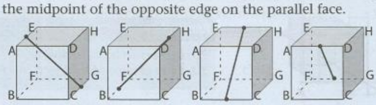
- Three axes of symmetry go through the midpoint of the two parallel faces; for example, ABCD and EFGH.



- Four axes of symmetry are drawn diagonally from a vertex to the opposite vertex on the parallel face.

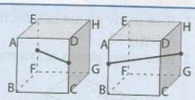


- Four axes of symmetry are drawn diagonally from the midpoint of an edge to the midpoint of the opposite edge on the parallel face.



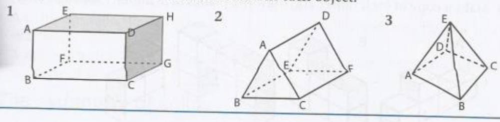
Worked example 3 (continued)

- Two axes of symmetry are drawn from the midpoint of and edge to the midpoint of the edge diagonally across from it.



Activity 3

Investigate the number of axes of rotation in each object.

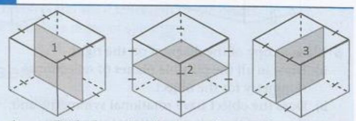


Determine plane symmetry

You have seen that a 2D-shape can have a line of symmetry along which the shape can be folded so that the two halves fit on each other. In the same way a 3D-object can have reflectional symmetry, but it has a plane of symmetry and not a line of symmetry. A plane of symmetry is a flat 2D-shape.

Worked example 4

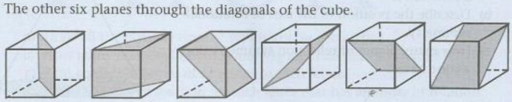
The cube in the diagram has side lengths of 2 cm. Planes 1, 2 and 3 divide the cube into two symmetrical parts in three different ways.



There are six other planes about which the cube also has reflectional symmetry. Find these planes.

Answers

The other six planes through the diagonals of the cube.

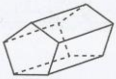


Activity 4

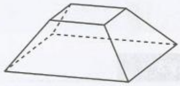
1 How many planes of reflectional symmetry are there in each shape in Activity 3?

2 How many planes of symmetry are there for each object?

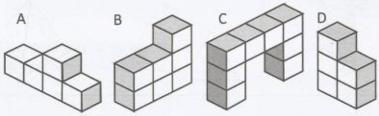
a)



b)

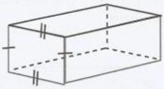


3 Make a copy of each object and draw the planes of symmetry for each one.

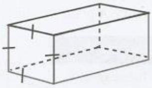


4 Investigate how many planes of symmetry each object has.

a) a cuboid (rectangular prism) with no square faces



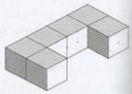
b) a cuboid with two opposite square faces



5 Make a copy of the diagram on the right.

a) Draw in all the possible planes of reflectional symmetry for the object.

b) Does the object have rotational symmetry, and, if so, give the axis of rotational symmetry.



6 On the right is the diagram of a cone.

a) How many axes of rotational symmetry does a cone have?

b) Describe the position of the axis of rotational symmetry.

c) How many planes of reflection symmetry does a cone have?

Summary

The order

- A shape has
shape.
- The order or
coincides wit
- A shape has
symmetry.
- A shape or ob
an angle of 18

The symme

- A 2D-shape m
of rotation.
- The order of r
be rotated thr
same as the or

Plane symm

- A 2D-shape ma
of symmetry a
- An object can

Revision e

1 Make a copy

T

a) Draw all th
b) How many
c) Find a poss
d) Give the or

Summary, revision and assessment

Summary

The order of rotational symmetry

- A shape has rotational symmetry if the rotated image looks like the original shape.
- The order or rotational symmetry is the number of times the image of a shape coincides with the original shape within a full turn of 360°.
- A shape has reflectional symmetry if it can be folded in half over the line of symmetry.
- A shape or object has point symmetry if it is unchanged by a rotation through an angle of 180°.

The symmetry of solids

- A 2D-shape may have a centre of rotation, but a 3D-object would have an axis of rotation.
- The order of rotational symmetry for a 3D-object is the number of times it can be rotated through an angle not greater than 360°, so that the image looks the same as the original object before the rotation.

Plane symmetry

- A 2D-shape may have a line of symmetry, but a 3D-object would have a plane of symmetry along which you could see a reflection of the object.
- An object can have more than one plane of symmetry.

Revision exercises

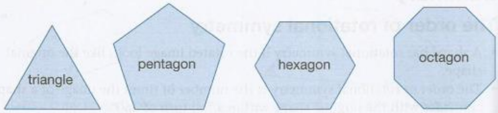
1 Make a copy of each letter below.

T H S A

a) Draw all the possible lines of symmetry for each letter.
b) How many possible lines of symmetry are there for each letter?
c) Find a possible centre of rotation for each letter.
d) Give the order of rotation for each letter that has an order of rotation.

Revision and assessment (continued)

2 Use the shapes to help you complete a copy of the table.

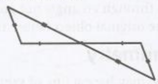


Shape	Order of rotational symmetry	Number of lines of symmetry
Triangle		
Pentagon		
Hexagon		
Octagon		

3 Give the order of rotational symmetry for each shape.

a)

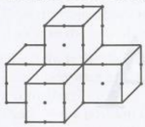
b)



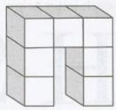
4 Make a copy of each shape in question 3 and draw all its possible lines of reflectional symmetry.

5 Make a copy of each diagram. Draw all the possible planes of reflectional symmetry for each object.

a)



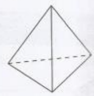
b)



6 Does each object in question 5 have rotational symmetry? If so, indicate its axis of rotational symmetry.

7 Use the diagrams

a)



a)	Tetrahedron equilateral
b)	Pentagonal a regular pen
c)	Square pyram a square)
d)	Cylinder

Assessment e

1 The word WHAT is

a) Which letters in

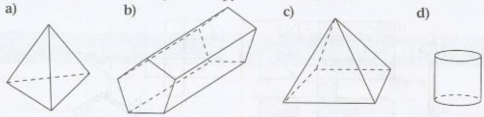
b) Which way in wh of symmetry?

c) Which letters in

d) Give the order of answer to questio

7 Use the diagrams to complete a copy of the table below.

a) b) c) d)



	Object	Number of rotational axes	Number of planes of symmetry
a)	Tetrahedron (four faces are equilateral triangles)		
b)	Pentagonal prism (the base is a regular pentagon)		
c)	Square pyramid (the base is a square)		
d)	Cylinder		

Assessment exercises

1 The word WHAT is set in two different ways (horizontally and vertically).

WHAT

W
H
A
T

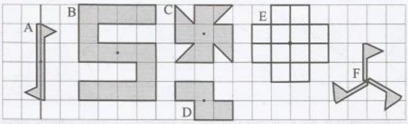
a) Which letters in the word have a line of symmetry?

b) Which way in which the word is set (horizontally or vertically) has a line of symmetry?

c) Which letters in the word have rotational symmetry?

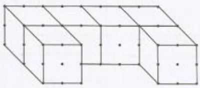
d) Give the order of rotational symmetry for the letter(s) you identified in answer to question 1c).

2 Make a copy of each shape.



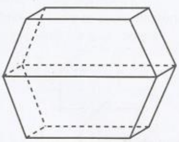
a) Investigate which shapes have rotational symmetry and give the order of rotational symmetry if applicable.
b) Investigate which of the shapes (if any) have a line of reflectional symmetry and insert the lines on your copy of the shape.

3 Make a copy of the object below.



a) Draw all the possible planes of reflectional symmetry for the object.
b) Does the object have rotational symmetry? If so, draw its axis of rotational symmetry.

4 Look at the diagram below.



a) How many planes of reflectional symmetry does the object have?
b) Investigate whether the object has any axes of rotation. If so, how many axes of rotation does it have?
c) Make a copy of the diagram and show the position of each plane of reflectional symmetry and the axes of rotational symmetry.

TOPIC 10

Sub-
Functions on a calc
Basic components
Algorithms
Methods of impleme

Starter activity

1 Use a calculato
a) 2 × 2 × 2 × 2
c) $\sqrt{729}$

2 Work in pairs.
a) Make a list o in the photo

b) Discuss the d

TOPIC

10 Computer and calculator

Sub-topic	Specific Outcomes
Functions on a calculator	• Demonstrate the use of different functions on a calculator
Basic components of a computer	• Describe the components of a computer
Algorithms	• Describe various methods of implementing an algorithm
Methods of implementing an algorithm	• Outline problem-solving stages

Starter activity

1 Use a calculator to find each value.

a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

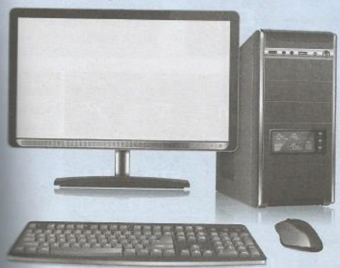
b) $\sqrt{441}$

c) $\sqrt{729}$

d) $\frac{22}{7}$

2 Work in pairs.

a) Make a list of the main parts of a computer. Identify the computer parts in the photograph.



b) Discuss the difference between computer software and hardware.

SUB-TOPIC 1

Functions on a calculator

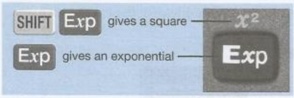
Demonstrate the use of different functions on a calculator

Nowadays a calculator is a necessary tool to make calculations easier to perform. A scientific calculator has higher mathematics functions for those who study algebra, trigonometry, and other branches of advanced mathematics. There are several brands of scientific calculators on the market. Make sure you study the instruction manual of your calculator so that you can use it properly.

Turning a calculator on and off

You should know how to turn a calculator on and off. On some scientific calculators there is a separate on/off key while you use SHIFT and AC (all clear) to turn other calculators on and off. Many calculators will shut off automatically when they are not used for a certain period of time. This saves battery power.

Most buttons have been programmed for more than one calculation. The second calculation is activated by pushing a button (usually SHIFT). The first calculation is usually marked on the button and the second calculation is printed immediately above it. For example, when you press sin, you will get the sin function of an angle. If you press SHIFT sin and then the value, you will get the angle for the sin value you entered.



Note

Using the shift key with another key does not necessarily give the same function on all calculators.

Note

You will learn to use the sin, cos and tan functions on a calculator when you learn about trigonometry.

Calculating

You can add frac
convert fraction

Worked exa

1 Using a sci
2 Write as a d
3 Calculate t
a) $\frac{528}{150 - 142}$

Answers

1 Press ÷ a
move to the
denominato
÷ and en
and enter th
denominato
To get a mix
gives $2\frac{59}{84}$.
You could al
2 To write $\frac{27}{54}$ a
key to toggle
calculators it
3 a) Do the fo
b) Enter the
calculation
c) You can us
denominat
Answer: 2

Activity 1

1 Use a calculat
a) 607 ÷ 1.86
d) $4\frac{1}{5} - 3\frac{2}{5}$
2 Use a calcula
a) $7\frac{4}{9}$
3 Use a calcula
a) 0.36

Calculating fractions

You can add fractions on some scientific calculators without having to first convert fractions to decimal fractions.

Worked example 1

1 Using a scientific calculator, calculate: $3\frac{3}{4} + \frac{2}{7} - 1\frac{1}{3}$

2 Write as a decimal fraction: $\frac{27}{54}$

3 Calculate the answers.

a) $\frac{528}{150 - 142}$ b) $150 - 90 \div 45 - 15$ c) $\frac{150 - 90}{45 - 15}$

Answers

1 Press [÷] and [SHIFT], then press [÷] again and enter the whole number; move to the position of the numerator and enter it; move down to the denominator position and enter the denominator; press [+]; and then [÷] and enter $\frac{2}{7}$ in the same way; press [−], then [SHIFT], then [÷] again and enter the last number, first the integer and then the numerator and denominator. Then press [=]. Answer: $\frac{227}{84}$

To get a mixed number, look for the shift key that indicates [$a^b/_c$] (d/c), which gives $2\frac{59}{84}$.

You could also do it this way: $(3 + 3 \div 4) + (2 \div 7) - (1 + 1 \div 3) = \frac{227}{84}$

2 To write $\frac{27}{54}$ as a decimal number, enter $27 \div 54$ [=] and then press the key to toggle between a common faction and a decimal number (on some calculators it is [S↔D]). The answer is 0.5, which we know is correct.

3 a) Do the following: $528 \div (150 - 142) = \frac{129}{4}$; as a decimal: 32.25

b) Enter the expression just as it is, the calculator will keep the order of calculations and do division first. Answer: 133

c) You can use [÷] and enter 150 – 90 as the numerator and 45 – 15 as the denominator, or you can use brackets as follows: $(150 - 90) \div (45 - 15)$. Answer: 2

Activity 1

1 Use a calculator to find the answers.

a) $607 \div 1.86$ b) $42.3 \times 3.97 \div 1\,635$ c) $2.1 + 3.42 \times 7.01$

d) $4\frac{1}{5} - 3\frac{2}{5}$ e) $\frac{5}{12} + \frac{2}{5}$ f) $3\frac{1}{5} \times 2\frac{1}{4}$

2 Use a calculator to change each fraction into a decimal number.

a) $7\frac{4}{9}$ b) $1\frac{9}{20}$ c) $3\frac{21}{25}$

3 Use a calculator to change each decimal fraction into a common fraction.

a) 0.36 b) 0.125 c) 0.375

Note

You will learn to use the sin, cos and tan functions on a calculator when you learn about trigonometry.

Working with exponents

On a scientific calculator there is usually a key marked [x^2] and the keys [x] or [y^x] or [x^y]. The last three show the same function: a number raised to any power. On some calculators, there is also a key marked [x^3] for finding the cube of a number.

Worked example 2

Use a calculator to find the answers.

1 15^2 2 5^3 3 2^7

Answers

1 Steps: enter 15, press [x^2], press [=]; answer: 225
2 Steps: enter 5, press [x^3], press [=]; answer: 125
3 Steps: enter 2, press [$\bar{x}$], press [Exp] 7, press [=]; answer: 128

On some calculators, there is also a key marked [x^{-1}] and on others, it is marked [$^1/_x$]. Both keys have the same function: they invert a number. For example, if the key sequence is [2] [x^{-1}] [=], the answer will be 0.5 or $\frac{1}{2}$. In other words, the key gives the reciprocal of the input value.

Activity 2

1 Use a calculator to find the answers.
  a) 16^2 b) 27^2 c) 182^2
  d) 221^2 e) 500^2
2 Calculate the answers.
  a) 4^3 b) 7^3 c) 10^3
  d) 15^3 e) 4.5^3
3 Calculate the answers.
  a) 2^{10} b) 2^{15} c) 3^6
  d) 5^5 e) $(2.1)^4$
4 Calculate the answers.
  a) $14^2 \times 7^2$ b) $25^2 + 20^2$ c) $7^3 + 5^3$
5 Calculate the volume of a cylinder using the formula $V = \pi r^2 h$ for the following values of r and h.
  a) $r = 5$ cm, $h = 7$ cm   b) $r = 5$ cm, $h = 15$ cm   c) $r = 3$ cm, $h = 6$ cm
6 Calculate the area x^2 of a square with the following side lengths for x.
  a) 15 cm   b) 5 m   c) 7 cm
7 Write down the value of each number.
  a) 25^{-1} b) 4^{-1} c) 2^{-1}
8 Write the answers for question 7 as decimal numbers.

Working w

Finding a squa
example, the s
by itself to give
calculators will
marked [$\sqrt[3]{x}$] or
There is also
for example $\sqrt[5]{2}$
root to find is ca
most calculators

Worked exa

Calculate the a
1 $\sqrt{169}$
2 $\sqrt[3]{512}$
3 $\sqrt[5]{243}$

Answers

1 Steps: press
answer: 13
2 Steps: press
the number,
3 Steps: press
root, then p

Activity 3

Round off each an
1 Use your calcu
a) $\sqrt{441}$
c) $\sqrt{32}$
e) $\sqrt{2\,500}$
2 Calculate th
a) $\sqrt[3]{729}$
c) $\sqrt[3]{27}$
e) $\sqrt[3]{45}$

Working with square, cube roots and other roots

Finding a square root means inversing the process of finding the square. For example, the square root of $15^2 = 225$. So, 15 is the number you have to multiply by itself to give 225. The key marked [$\sqrt{x}$] on some calculators or [$\sqrt{\square}$] on other calculators will give you the square root of a number. In the same way, the key marked [$\sqrt[3]{x}$] or [$\sqrt[3]{\square}$] will give you the cube root of a number.

There is also the key [$\sqrt[\blacksquare]{\square}$] with which you can calculate the root of any number, for example $\sqrt[5]{243}$, the fifth root of 243. The small number that tells you which root to find is called the order of the root. The root $\sqrt[5]{243}$ is of the fifth order. On most calculators, use [SHIFT] and [$x^{\blacksquare}$].

Worked example 3

Calculate the answers.

1 $\sqrt{169}$
2 $\sqrt[3]{512}$
3 $\sqrt[5]{243}$

Answers

1 Steps: press [$\sqrt{\square}$], then enter the number, then [=] and read off the answer: 13
2 Steps: press [$\sqrt[3]{\square}$] (on some calculators it is the shift key of [$\sqrt{\square}$]), then enter the number, then press [=] and read off the answer: 8
3 Steps: press [$\sqrt[\blacksquare]{\square}$] ([SHIFT] [$x^{\blacksquare}$]), then enter the number and the order of the root, then press [=] and read the answer: 3

Activity 3

Round off each answer to two decimal places where necessary.

1 Use your calculator to find the answers.

a) $\sqrt{441}$ b) $\sqrt{1\ 444}$
c) $\sqrt{32}$ d) $\sqrt{15}$
e) $\sqrt{2\ 500}$

2 Calculate the answers.

a) $\sqrt[3]{729}$ b) $\sqrt[3]{1\ 728}$
c) $\sqrt[3]{27}$ d) $\sqrt[3]{3\ 375}$
e) $\sqrt[3]{45}$

Activity 3 (continued)

3 Find the anwers.

a) $\sqrt[9]{512}$ b) $\sqrt[4]{1\,728}$

c) $\sqrt[4]{81}$ d) $\sqrt[4]{19.4481}$

e) $\sqrt[6]{4\,096}$

4 Use a calculator to find the answers.

a) $(2\sqrt{5} + \sqrt{5})^2$ b) $\sqrt{10} \times \sqrt{40}$

c) $\frac{-3 + \sqrt{14}}{-2}$

5 If $a = -1.2$; $b = 4.5$; and $c = -4$, simplify the following.

a) $\sqrt{a^2}$ b) $\sqrt{b^2 - 4ac}$

c) $\frac{b^2 - \sqrt{b - c}}{2a}$ d) $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$

Scientific notation on the calculator

When we work with very large or very small numbers, it is useful to use scientific notation.

You worked with scientific notation in previous grades. Remember that we can write numbers like 0.0000003695 as:

$$3.695 \times \frac{1}{10\,000\,000} = 3.695 \times \frac{1}{10^7} = 3.695 \times 10^{-7}.$$

We can write a large number like 5 800 000 as $5.8 \times 1\,000\,000 = 5.8 \times 10^6$.

A scientific calculator has a key you can use to enter the exponent of the second part of the number, the power of 10. In this case, it is 10^6. It is either marked Exp or ×10x or in some similar way. In other words, using Exp will allow the numbers you enter to be registered as powers of 10.

Worked example 4

Use a calculator to multiply 3.8×10^5 by 2.4×10^{-3}.

Answer

In this answer, we will assume that the calculator has the key ×10x that you can use to enter the powers of 10.

Key in numbers in sequence:

$3.8 \times 10^5 \times 2.4 \times 10^{-3}$
$= 91\,200$
$= 9.12 \times 10^4$

There are also other key sequences you can use to find the correct answer.

Activity 4

1 Use a calculator
a) (1.25×10^7)
b) (2.75×10^{-9})
c) (4.33×10^4)
d) (1.75×10^{-3})

2 Calculate: (4.5 × … / (8.9 × …

3 The mass of a p…
a) Find the mas…
Remember, 1…
b) How many p…

Finding prime …

On some calculators, …
For example, 75 = 3 × …

Worked example …

Find the prime facto…
1 42
2 725

Answers

Use the following ke…
The display will sho…
(SHIFT of …) the di…
1 $42 = 2 \times 3 \times 7$
2 $725 = 5^2 \times 29$

Activity 5

Write down the prime …
1 2 250
3 3 240
5 11 025
7 108

Activity 4

1 Use a calculator to find the answers.

a) $(1.25 \times 10^7) \times (3.15 \times 10^6)$

b) $(2.75 \times 10^{-9}) \times (1.6 \times 10^{-4})$

c) $(4.33 \times 10^4) + (2.65 \times 10^5)$

d) $(1.75 \times 10^{-3}) - (1.6 \times 10^{-4})$

2 Calculate: $\frac{(4.5 \times 10^{-3} + 3.7 \times 10^{-2})}{(8.9 \times 10^{-3} + 3.1 \times 10^{-2})}$

3 The mass of a pin is 1.07×10^{-4} kg.

a) Find the mass of a pin in grams.
Remember, 1 kg = 1 000 g.

b) How many pins are in one kilogram of pins?

Finding prime factors of a number

On some calculators, there is a key that gives the prime factors of a number. For example, $75 = 3 \times 5^2$. On some calculators, use [SHIFT] combined with [° ' "].

Worked example 5

Find the prime factors of the following numbers.

1 42
2 725

Answers

Use the following key sequence: enter the number and then press [=]. The display will show 42. Then press [SHIFT] followed by the FACT key ([SHIFT] of [° ' "]) the display will show the answers.

1 $42 = 2 \times 3 \times 7$
2 $725 = 5^2 \times 29$

Activity 5

Write down the prime factors of each number in exponential notation.

1 2 250	2 546
3 3 240	4 756
5 11 025	6 120
7 108	8 504

SUB-TOPIC 2

Basic components of a computer

Describe components of a computer

Almost everyone has seen a computer and knows that it is an electronic device people use to handle many tasks quickly. Examples of the tasks a computer can do include storing information and doing calculations very fast, referencing and cross-referencing information, finding stored information and processing information (data). Computers have also become devices for immediate contact through email and the internet (online services). We can do almost anything with the help of computers. Scientists and researchers are constantly finding new applications for computers in the fields of medicine, engineering, banking, education, agriculture and so on.

When you buy a computer, you buy a few big pieces that take up space on a table or a desk. These are called computer **hardware**. In order to make a computer work when it is switched on, you need **software**. Anything you buy to use with your computer is either hardware or software.

Did you know?

The first electronic computer was developed in 1946. It was called the ENIAC (Electronic Numerical Integrator and Computer) and took up 1 800 square feet and weighed 30 tons.

New words

hardware: the physical components of a computer
software: the programs that are used by a computer

Activity 6

1 Make a list of all the different, necessary physical parts of a basic desk computer.
2 Write down at least three instances in life, other than in the home, where computers are used.

Computer hardware

Computer hardware includes the central processing unit case, the monitor, the mouse and the keyboard.

Computer monitor

The computer monitor is the part of the system on which text and images are displayed.
There are two main types of monitor:
- The older type is a cathode ray tube design (CRT).

A cathode ray tube monitor

- The second typ
or LED. An exa
alongside.
LED stands for
LCD stands for

Computer cas

A computer case c
that make a comp
also known as a c
Cases come in ma
The size and shap
determined by th
motherboard, sinc
of most computer
many of the impo
of the system, suc
unit (CPU) and m
connectors for per
devices that are co
not part of it, such

Computer case

A computer case co
(central processing
brain of the comp
the RAM (random
stores informatio
use while it is swi
erased when a co

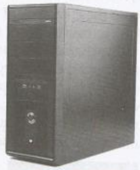
The computer
can be used to co
devices. These op
example, printers
computers via US
connected to com
keyboard, the mo

- The second type is a flat panel design, also called LCD or LED. An example of a flat panel monitor is shown alongside.
  LED stands for light-emitting diodes.
  LCD stands for liquid-crystal display.


LED monitor

Computer case

A computer case contains all the components that make a computer work. A computer case is also known as a computer chassis or tower. Cases come in many different sizes and designs. The size and shape of a computer is usually determined by the size and shape of the motherboard, since it is the largest component of most computers. The motherboard holds many of the important electronic components of the system, such as the central processing unit (CPU) and memory. It also provides connectors for peripherals. Peripherals are devices that are connected to the computer, but not part of it, such as the keyboard and mouse.


Computer case


Computer case


Computer case opened to expose components

A computer case contains parts such as the CPU (central processing unit), the micro-processor or brain of the computer, the power supply and the RAM (random access memory). The RAM stores information that the computer needs to use while it is switched on. RAM information is lost when a computer is switched off.

The computer case has many openings that can be used to connect a computer to other devices. These openings are called ports. For example, printers are usually connected to computers via USB ports. Other devices that are connected to computers via the ports are the keyboard, the mouse and speakers.


The power connection and different ports are visible on the back of this computer case.


Device plugged into a USB port

The keyboard

We use a keyboard to input text into a computer. There are many different types of keyboard (see the photograph). In the basic design of the most common keyboard, the keys are arranged according to the QWERTY layout. This name comes from the characters of the first six keys across the top of the keyboard.

The characters are not made by the keys on a keyboard (as was the case with typewriter keys), but when you press a key, it inputs a value that the computer translates into a character on the screen.

Three different computer keyboards

◀ Letters are printed by typewriter keys. This photograph shows the back of the keys for a few letters, the number 2 and quotation marks.

The mouse

The handheld device that you use to point-and-click is called a mouse, because it looks a little like a mouse. There are several types of mouse. Some are plugged into the computer and use a wire to connect while others are wireless and connect remotely.

The mouse is used to move the cursor, scroll through text, find a position in text quickly, and so on.

A wired mouse and a wireless mouse

Storage

Inside a computer case, there is a hard disk drive on which information is stored permanently. It can hold massive amounts of information such as all the programs on a computer and all the data files.

Almost all computers nowadays have a drive for a CD or a DVD. These drives can also write to a CD or DVD. That means transferring information from the computer to the disc.

The CD/DVD drive is built into the computer tower.

You can use C
storage units fo
regular back-up
of the files and
Flash drives a
have to store, b
Every year they
store more infor
drives of up to 5
available.

Computer s

If you use a com
If you could not
name software d
make it possible
Nowadays, advan
(from typing to d
Modern progra
need logical instr
computer can ca
a word-processin
computer. We ca
into the codes th
translated back to

Activity 7

Identify the parts
each part.

You can use CDs and DVDs as external storage units for your files. It is wise to make regular back-up copies on CDs or flash drives of the files and data on your hard drive.

Flash drives are very handy when you have to store, back-up and transfer data. Every year they become cheaper and can store more information. In January 2014 drives of up to 512 gigabytes (GB) were available.

It is convenient to store and transport information on a flash drive.

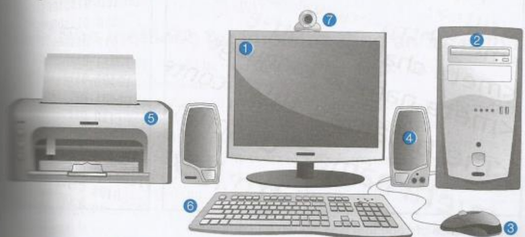
Computer software

If you use a computer, you have to be able to communicate with the computer. If you could not communicate with a computer, it would be of no use to you. The name software describes the programs we load onto computers. These programs make it possible for us to enter information on a computer and use the computer. Nowadays, advanced programs that allow us to do many things on computers (from typing to drawing and embroidery) are available.

Modern programming languages are complicated and many-layered. Computers need logical instructions that have to be written into number codes so that a computer can carry out instructions. This is why we have to load programs such as a word-processing program that can act as an interpreter between a person and a computer. We can type text in the usual way and the program translates the input into the codes that the CPU can handle. After computing the codes, they are translated back to text that we can read.

Activity 7

Identify the parts of the computer below and briefly describe the purpose of each part.



SUB-TOPIC 3 Algorithms

Introduction

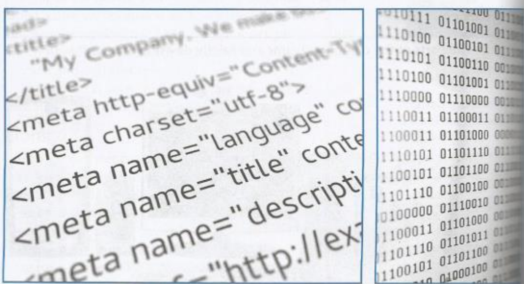
In the previous section, you saw that a computer cannot work unless it is given coded instructions. Most people do not understand the machine code that a computer needs so that it can run a program. Therefore programmers write the instructions in a programming language such as Pascal or C. These instructions are then changed into computer code or machine code.

Programmers have to know exactly what a computer is supposed to do. Remember, a computer cannot do anything that it has not been programmed to do. It cannot think that there is a mistake and that it needs to correct something. If a program is not clear and correct, the computer will not do what you want it to do or it will not work at all.

An **algorithm** is a set of instructions (much like a recipe) that is used to solve a particular problem in a step-by-step procedure. It describes precisely what has to be done with **input** to get **output** and it stops after a time. This means that there has to be a result.

> **New words**
>
> **algorithm:** a set of operations that produces a result
> **input:** information that is needed to solve a problem
> **output:** the result of a calculation

An algorithm is not the code that is fed into a computer. It is simply a list of the instructions that gives information about how to write computer code. The first step in designing an algorithm is usually to make up a flow chart or pseudo code. You will learn more about these concepts later in this topic.



Two examples of code that was written for computers.

Worked examp

Work in small grou
Arrange the series o
dial a number; stop
conversation; lift re

Answer

We usually write an
Start; look up a num
replace receiver; sto

Activity 8

1 The following in
order.
wake up
get dressed
drink tea
2 Design an algorit

The characteristi

Note the following ch
- It should be clear a
logically and no ste
- The instructions fo
immediately unders
- The instruction sho
finite number of ste
- An algorithm shoul

Describe met

The following method
- Define the problem
- Write down what th
will be. The outputs
- Write down the steps
output in the correct
which steps must be
other steps can be ca
- Choose data and test
how well it works.

rk unless it is given
hine code that a
grammers write the
. These instructions are

upposed to do.
been programmed to
to correct something.
do what you want it to

New words

thm: a set of operations that
es a result
information that is needed
e a problem
: the result of a calculation

. It is simply a list of the
mputer code. The first
chart or pseudo code.

Worked example 5

Work in small groups or in pairs.
Arrange the series of words below to form meaningful instructions for an activity.
dial a number; stop; replace receiver; start; look up a number; have a conversation; lift receiver

Answer

We usually write an algorithm as follows:
Start; look up a number; lift receiver; dial a number; have a conversation; replace receiver; stop.

Activity 8

1 The following instructions have been jumbled. Arrange them in a logical order.

wake up	walk to school	make your bed
get dressed	clean your teeth	wash the dishes
drink tea	take a bath	eat breakfast

2 Design an algorithm for converting marks out of 50 to percentage.

The characteristics of an algorithm

Note the following characteristics of an algorithm:

- It should be clear and well ordered. This means that the steps should follow logically and no steps should be left out.
- The instructions for the operations should be clear so that a programmer immediately understands what was intended.
- The instruction should be given in such a way that they can be carried out in a finite number of steps.
- An algorithm should have a start and a finish.

Describe methods for implementing an algorithm

The following methods should be used when implementing an algorithm:

- Define the problem clearly and as briefly as possible.
- Write down what the inputs will be (information needed) and what the outputs will be. The outputs show the result that the algorithm should give.
- Write down the steps that are needed to convert the input to the required output in the correct **sequence**. The order of steps is important as it shows which steps must be carried out before other steps can be carried out.
- Choose data and test the algorithm to see how well it works.

New word

sequence: a structure in computing where statements are executed (carried out) one after the other

Worked example 6

Write an algorithm to calculate the total of the marks and the average mark obtained in six subjects of which all are given out of a total of 300.

Answer

The total of all the marks will be $6 \times 300 = 1\ 800$

1 Start.	2 Let the sum be 0.
3 Find the first mark.	4 Add first mark to sum.
5 Find the second mark.	6 Add second mark to sum.
7 Find the third mark.	8 Add third mark to sum.
9 Find the fourth mark.	10 Add fourth mark to sum.
11 Find the fifth mark.	12 Add fifth mark to sum.
13 Find the sixth mark.	14 Add sixth mark to sum.
15 Divide sum by 1 800.	16 Output the average.
17 Stop.	

The problem with this version of the algorithm is that it is long because steps are repeated. We could make it shorter by changing it to the version below. We enter the marks as the values and get a value after each addition time and add it to the sum. However, we want it to stop when all the values have been added and so we include –1 as a last (seventh) value and use it to terminate the addition.

1 Start.
2 Let the sum be 0.
3 Find a value.
4 If the value equals –1, go to step 7.
5 Let sum = sum + value.
6 Go to step 3 to find the next value.
7 Divide sum by 1 800.
8 Output the average.
9 Stop.

Flow charts are a useful tool to use when trying to achieve the characteristics of an algorithm. In the next sub-topic, you will learn about flow charts.

Activity 9

Work in groups of four learners. Each group then divides into two teams (A and B). Follow the instructions.

Team A uses the first algorithm in Worked example 6 to work out the total for the marks below. Team B uses the second method. See which team finds the answer quicker. Swap methods for number 2. Write down the steps.

1 149; 203; 155; 164; 96; 134
2 171; 84; 144; 76; 190; 162

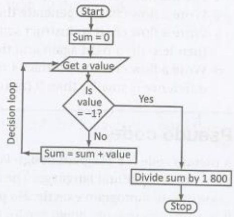
SUB-TOPIC 4 Methods of implementing an algorithm

Flow charts and decision loops

A flow chart shows the steps in a process. It consists of boxes (rectangles), diamonds (rhombi), parallelograms and other shapes that are connected with arrows. Each shape represents one step in a process. The arrows that link shapes show the order of the steps in the algorithm.

An action is written in a rectangle (a box); input is written in a parallelogram and a diamond is used to write a question or information that leads to a decision.

Sometimes a decision box results in a loop back to a previous step. The flow chart shows the algorithm in Worked example 6. The decision loop tells the computer that there is a calculation that needs to be done repeatedly. The decision loop originates in a decision box (a diamond-shaped box).

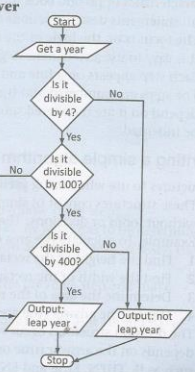


Worked example 7

Write a flow chart to determine whether a particular year is a leap year or not.

Use this information: The year number of a leap year is divisible by 4. If a year number is divisible by 4, but not by 100, the year is a leap year. If a year number is divisible by 4 and by 100, but not by 400, the year is not a leap year. If a year number is divisible by 4, by 100 and by 400, it is a leap year.

Answer



Activity 10

1. Use the flow chart of Worked example 7 to determine whether the following years are/were leap years.

a) 1964	b) 1970	c) 2014	d) 2020
e) 1600	f) 1700	g) 1800	h) 1900

2. Write a flow chart to determine whether a number is an odd number or an even number.
3. Use the flow chart in question 2 to test the numbers between 81 and 99.
4. Write a flow chart to generate the multiples of 3 between 0 and 100.
5. Write a flow chart to instruct someone about how to cross a road (look right, then left, then right again and then cross the road if it is safe to do so).
6. Write a flow chart to subtract 4 repeatedly, starting with 30, until the difference is smaller than 0 (zero).

Pseudo code

A pseudo code is a program design language that is made up of statements that are written in natural language. The design language describes the steps for the algorithm of a program exactly. No programming code appears in pseudo code. It is only used to write down the logical steps in a process and to write code for a program.

Characteristics of pseudo code include:

- All statements describe actions to be taken.
- The focus is on the logic of the algorithm.
- It is easy to use a statement to generate the programming code.
- Each step appears on a line and only one step is written on one line.
- The steps are numbered and the statements in selections and repetitions that depend on it are numbered subordinately (for example, 1.1). They can also be indented.

Writing a simple algorithm in pseudo code

Structures to use when using pseudo code include:

- These structures consist of simple steps that are carried out one after the other without loops or decisions. These steps are called sequences.
  Example: To calculate the area of a rectangle:
  1 Find the height of the rectangle.
  2 Find the width of the rectangle.
  3 Determine the area of the rectangle using the formula, height × width.
  4 Display the answer (the area).
- The decision structures occur when a question is asked and the next step depends on the answer (true or false). In this case words (all capital letters), such as IF, THEN, ELSE and ENDIF are used.

Example: To dete
1 IF number of
1.1 Print goo
2 ELSE
2.1 Print usu
3 ENDIF
- The loop structure
Words to use are
Example: To find
1 WHILE age is
1.1 print nam
2 ENDWHILE

Worked example

Use pseudo code to
quotient is 1. Write

Answer

1 Write inputs: di
2 Set divisor = 2
3 Set counter = 0
4 Get dividend.
5 Calculate quotie
6 WHILE quotient
6.1 Set divide
6.2 Set counte
7 ELSE
7.1 Print coun
8 ENDWHILE

Outline proble

When you have to solv
- **Understand the pro**
- **Devise a plan.** Take
goal (arrive at the re
- **Carry out the plan.**
changes if needed.
- **Look back.** Evaluate

Example: To determine who sold more cars than a certain quota:

1 IF number of cars sold is greater than quota THEN
   1.1 Print good sales message.
2 ELSE
   2.1 Print usual sales message.
3 ENDIF

- The loop structures are used when actions are repeated for a number of steps. Words to use are DO WHILE; DO UNTIL and ENDDO.
  Example: To find learners who are younger than 13 in a group:
  1 WHILE age is smaller than 13 THEN
     1.1 print name
  2 ENDWHILE

Worked example 8

Use pseudo code to write an algorithm to divide 128 by 2 repeatedly until the quotient is 1. Write a flow chart for this procedure.

Answer

1 Write inputs: dividend; divisor; quotient; counter.
2 Set divisor = 2
3 Set counter = 0
4 Get dividend.
5 Calculate quotient as dividend divided by divisor.
6 WHILE quotient is greater than 1 THEN
   6.1 Set dividend equal to dividend divided by 2
   6.2 Set counter = counter + 1
7 ELSE
   7.1 Print counter
8 ENDWHILE

Outline problem-solving stages

When you have to solve a problem, it is useful to follow these steps:

- **Understand the problem.** Make sure that you know what has to be determined.
- **Devise a plan.** Take all factors into consideration and find a way to reach the goal (arrive at the required result).
- **Carry out the plan.** Do what you planned in the previous step and make changes if needed.
- **Look back.** Evaluate the success of your plan; did it work as it should have?

When you plan to use a computer for a task such as calculating the average marks of a class, the problem is how to get the computer to do the task. In this case, you have to think carefully about all the steps that have to be followed. If you are not a programmer, you will have to ask someone to write a program for you, but you will need to give clear instructions about what you want done. Writing a flow chart for an algorithm can help you write clear instructions for the program.

The flow chart or algorithm (or a pseudo code) is the solution to your problem.

Activity 11

In each case, first explain clearly what has to be done.

1 Write a flow chart for the following problem.
   a) Starting with 512, divide by 2 repeatedly until the quotient is 1.
   b) Starting with 120, divide by 15 repeatedly until the quotient is 1, give further instructions.
2 Write a flow chart and a pseudo code for a program to find the mean of five numbers.
3 Write a pseudo code for a program to find the area of a circle with radius 7.
4 Write a flow chart for a program to find the volume of a cylinder, given the height and radius of base ($V = \pi r^2 h$).
5 Write a flow chart for an algorithm to calculate any power of a variable x.
6 Write a flow chart to find the quotient of a dividend divided by a divisor, keeping in mind that the divisor may not be 0.
7 Write a flow chart to calculate the surface area of a cylinder with height h and radius of base r.
8 Pythagoras' theorem states that in a right-angled triangle, the square of the hypotenuse equals the sum of the squares of the right-angled sides. Let the hypotenuse be c and the two right-angled sides be a and b. Write a flow chart to calculate the hypotenuse in a right-angled triangle if the other two sides are given.

Looking at a simple programming language

BASIC is a simple computer language with the following characteristics:
- Every line is numbered in multiples of 10, starting with 10.
- The last line contains only the command END.
- Capital letters are used throughout.

Example

The example b
uses the answe

```
10   Z = 9
20   FOR N =
30   Z = Z/4
40   Z = Z + 3
50   PRINT Z
60   NEXT N
70   END
RUN
```

Start with the v
computer to pe
previous value a

Activity 12

1 Write a flow
2 If possible,
3 Do this prog
   9 → ÷ 4 → +
   Compare yo
4 Repeat the c
5 Look at the
   a) Write a p
   b) Run the p
   c) Do the pr
   before. U
   30 → ÷ 5
   → ÷ 5 →
   Compare
   computer
   d) Change th
   and repeat
   i) 14.3
   ii) 15
   e) Change th
   above step
   f) What do y

Example

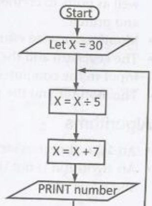
The example below shows a program that starts with 9, performs an operation and uses the answer to perform the same operations. This is repeated 25 times.

```
10    Z = 9
20    FOR N = 1 TO 25
30    Z = Z/4                (Z/4 means Z ÷ 4)
40    Z = Z + 3
50    PRINT Z
60    NEXT N
70    END
RUN
```

Start with the value for Z in line 10. The next line (line 20) is an instruction to the computer to perform the loop 25 times. Line 30 gives Z a new value $\frac{3}{4}$ of the previous value and line 40 adds 3 to that value.

Activity 12

1 Write a flow chart for the program in the above example.
2 If possible, run the program on a computer.
3 Do this program manually on your calculator. You can use the key sequence: 9 → ÷ 4 → + 3 = and then the answer is → ÷ 4 → + 3 = 25 times.
Compare your answer with the computer's answer.
4 Repeat the computer program with the numbers Z = 7 and Z = –11.
5 Look at the flow chart. Let the loop repeat 25 times.
  a) Write a program in BASIC for this flow chart.
  b) Run the program as for question 2.
  c) Do the program manually on your calculator as before. Use the key sequence: 30 → ÷ 5 → + 7 = and then answer → ÷ 5 → + 7 = ... 25 times.
  Compare your answer with the computer's answer.
  d) Change the input value X to the values below and repeat the steps (b) and (c) above.
    i) 14.3
    ii) 15
  e) Change the instruction X = X ÷ 5 to X = X ÷ 9 and repeat all the above steps.
  f) What do you notice about the output?



TOPIC 10

Summary, revision and assessment

Summary

Functions on a calculator

- A calculator performs rote calculations quickly.
- A calculator is only as accurate as the user.
- You can work with fractions in the common or improper form.
- You can work with fractions in decimal form.
- You can switch between decimal fractions and common fractions.
- You can work out percentages.
- You can work out powers of numbers.
- You can work out square roots, cube roots and any other root of a number.
- You can work out prime factors of a number.
- You can easily switch between scientific notation and the ordinary notation for numbers.

Basic components of a computer

- A computer can be programmed to perform complicated tasks.
- The most important part of a computer is the CPU (central processing unit).
- The case also contains the power supply, the hard drive and a DVD/CD drive as well as ports to connect external devices, such as a keyboard, mouse, speakers and printer.
- Monitors provide visual communication between users and computers.
- The keyboard and the mouse are the instruments with which a user provides input to the computer.
- The monitor and the printer provide the output to the user.

Algorithms

- An algorithm provides a way to structure the commands for a program logically.
- An algorithm is not the program in code; it is only an explanation of what should be done.

Methods of implementing an algorithm

- A flow chart is an aid to writing a well-structured algorithm and program.
- A flow chart breaks down the calculation or action into a sequence of simple steps that can be used to plan the writing of the program.
- It is easy to switch between scientific notation and ordinary number notation.
- There are many computer languages and every computer language has its own rules.

Revision

1 Use your c
three deci
a) $-\frac{23}{25}$
2 Use your c
fractions.
a) 0.48
3 Use your c
a) 16^2
e) 13^3
4 Use your c
a) $\sqrt{16}$
5 Write the v
a) 0.000008
b) 129 000
c) 0.000207
d) The sun
e) A certain
6 Name the m
7 Explain the
8 Write an alg
goods that a
9 Write a flow
of $3x + y = 15$
10 Write a flow
y-intercept of
11 Look at the f
answers for q

Revision exercises

1 Use your calculator to write the following as decimal fractions. Round off to three decimal places where necessary.

a) $-\frac{23}{25}$ b) $2\frac{8}{15}$ c) $-5\frac{13}{35}$ d) $1\frac{3}{11}$

2 Use your calculator to write the following decimal fractions as common fractions.

a) 0.48 b) 2.4 c) –31.003 d) 0.56

3 Use your calculator to calculate the following.

a) 16^2 b) 56^2 c) 117^2 d) 17^2
e) 13^3 f) 5^6 g) 9^4 h) $(12.4)^5$

4 Use your calculator to calculate the following.

a) $\sqrt{16}$ b) $\sqrt{12.79}$ c) $\sqrt[3]{125}$ d) $\frac{(1+\sqrt{5})}{2}$

5 Write the values in scientific notation.

a) 0.0000086
b) 129 000 000 000 000
c) 0.0002079
d) The sun is about 150 000 000 km from the earth.
e) A certain virus has a length of about 0.00038 millimetres.

6 Name the most important pieces of computer hardware.

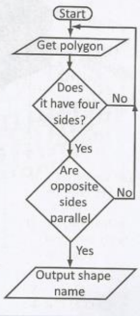
7 Explain the term software.

8 Write an algorithm and draw a flow chart to calculate the sale price of goods that are reduced by 11%.

9 Write a flow chart for a program to find the gradient of $3x + y = 15$.

10 Write a flow chart for a program to find the x- and y-intercept of $3x + y = 15$.

11 Look at the flow chart and write down the possible answers for questions in the flow chart.



Assessment exercises

1 Use your calculator to find the answers. Round off to three decimal figures where necessary.

a) $\sqrt{1.44}$ b) $\sqrt{12.96}$ c) $\sqrt[3]{91.125}$ d) $\sqrt{1\frac{7}{9}}$

e) $\frac{(2 + \sqrt{5})}{\sqrt{5}}$ f) $(2\sqrt{5} + \sqrt{5})^2$ g) $2 + \sqrt{64}$ h) $\sqrt{10} \times \sqrt{40}$

2 Write in scientific notation.

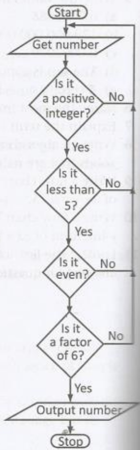
a) 317 000 b) 0.0000000046

c) 949 865 000 d) 543.42

3 Write an algorithm and draw a flow chart to sort out squares from a collection of shapes.

4 Write a pseudo code for a program to find the sum of the first 100 natural numbers.

5 Look at the flow chart. Write down the possible answers for the questions in the flow chart.



Answe

Topic 1 Sets

Starter activity

1 a) A = {Sunday Wednesday Saturday}
b) A = {2, 3, 5,
c) A = {tennis, Other answ
d) A = {footbal Other answ
e) Many differ Below is an A = {Kitwe, Kabompo,
2 Answers will di
3 a) Finite; there number (2).
b) Infinite; ther of prime nu

Activity 1

1 a) A = {the first
b) M = {the first where 8 is m of 10}
c) A = {the lette *January*}
2 A = {x: 100 < 2x
3 a) B = {2, 3, 5, 7,
b) D = {13, 26, 3
4 P = {x: 3 ≤ x ≤ 8,
or P = {x: 2 < x <
5 A = {girls in your in the school} or your class}
B = {4, 9, 16, 25}; of integers}
C = {Lusaka, Ndo Chinsali}; E = {all Zambia}
D = {all children i population of Zam in Africa}
6 a) $n = 7$
c) $n = 1$
7 a) False b) False

Answers to activities

Topic 1 Sets

Starter activity

1 a) A = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
 b) A = {2, 3, 5, 7, 11}
 c) A = {tennis, table tennis, cricket} Other answers may also be correct.
 d) A = {football, rugby, volleyball} Other answers may also be correct.
 e) Many different sets are possible. Below is an example.
 A = {Kitwe, Kabwe, Kasama, Kabompo, Kataba}

2 Answers will differ.

3 a) Finite; there is only one even prime number (2).
 b) Infinite; there is an infinite number of prime numbers.

Activity 1

1 a) A = {the first seven prime numbers}
 b) M = {the first four positive integers where 8 is multiplied by a power of 10}
 c) A = {the letters that spell the word *January*}

2 A = {x: $100 < 2x < 150$, $x \in \mathbb{N}$}

3 a) B = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}
 b) D = {13, 26, 39, 52, ...}

4 P = {x: $3 \le x \le 8$, $x \in \mathbb{N}$}
 or P = {x: $2 < x < 9$; $x \in \mathbb{N}$}

5 A = {girls in your class}; E = {all the girls in the school} or E = {all learners in your class}
 B = {4, 9, 16, 25}; E = {the set of squares of integers}
 C = {Lusaka, Ndola, Livingstone, Chinsali}; E = {all towns and cities in Zambia}
 D = {all children in Zambia}; E = {the population of Zambia} or E = {children in Africa}

6 a) $n = 7$ b) $n = 26$
 c) $n = 1$ d) $n = 0$

7 a) False b) False c) False d) True

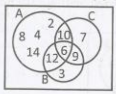
8 A = D (A, B, C and D are equivalent.)

9 a) {4, 5, 6} or any other three numbers between 3 and 10
 b) {d, e, f} or any other three letters in the alphabet that have not been listed

Activity 2

1 a) $A \cap B$ = {6, 12}
 b) $A \cap C$ = {6, 10}
 c) $B \cap C$ = {6, 9}
 d) $A \cap B \cap C$ = {6}

2

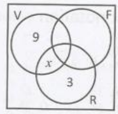


3 a) $A \cap B$ = {10, 12}
 b) $A \cup B$ = {6, 8, 9, 10, 11, 12, 13, 14}

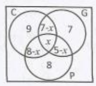
4 a) P = {a, b, d, g}
 b) Q = {c, d, e, f, i, l}
 c) R = {c, f}
 d) $P \cup Q$ = {a, b, c, d, e, f, g, i, l}
 e) $Q \cap R$ = {c, f}
 f) $(P \cup Q)'$ = {j, k}

Activity 3

1 $x = 1$



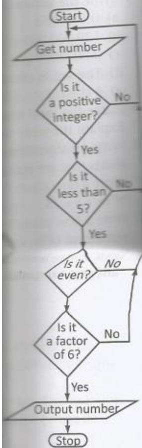
2 16



a) Two people like all three types of soap.
b) 14 people like only two types of soap.

Activity 4

1 A 2 $(A \cup B)'$
3 $(A \cap B) \cup C$ 4 $B \cap (A \cup C)'$



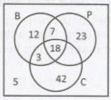
Activity 5

1 19 learners were tested, and 16 were not tested for HIV or for TB.

2 a) 70 delegates b) 59 delegates

3 12 people

4

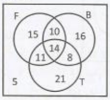


a) 120 – (12 + 7 + 23 + 13 + 18 + 42) = 120 – 115 = 5 students

b) $n(P \cap B \cap C') = 7$ students

5 No student ordered neither cake nor salad.

6



a) $(F \cup B)' \cap T = 21$

b) $F \cup T \cap B' = 15 + 11 + 21 = 47$

c) $(F \cup B \cup T)' = 100 - (15 + 11 + 21 + 8 + 14 + 10 + 16) = 100 - 95 = 5$

7 a) x represents the number of learners who chose red and blue; $x = 192$

b) 160 learners

8 18 people

Topic 2 Index notation

Starter activity

1 a) $2 \times 2 \times 2$ b) 5×5 c) 7×7

2 a) 2^4 b) 3^3 c) 6^5 d) 11^3 e) 5^6 f) 7^4

3 a) 2^5 b) 2^{12}

Activity 1

1 a) 5^2 b) 5^4 c) a^5 d) 2^3 e) 12^3 f) $3^2 \times b^4$

2 a) 2 b) 0 c) 4 d) 3 e) 2 f) –1

3 a) a b) b c) 3 d) 15 e) x f) 9

4 a) $a \times a \times a \times a$ b) $c \times c$ c) $2 \times 2 \times 2 \times 2 \times 2$ d) $10 \times 10 \times 10 \times 10$ e) $ab + ab + ab$ f) $x + x + x + y \times y \times y \times y \times y \times y$ g) $5 \times 5 \times 5 \times 2 \times 2$ h) $4 \times 4 \times 4 \times x \times x \times x$ i) $2 \times x \times 2 \times x \times 2 \times x$

Activity 2

1 a) 2^3 b) 2^{10} c) $3^2 \times 2^2$ d) $10a^5$ e) a^3 f) 6^4 g) 5^{-n} h) 2^0 i) $-3 \times 2^2 \times b^6$

2 a) m b) m^{-2} c) $(xy)^2$ d) m^1 e) $p^0 = 1$ f) a^3

3 a) m^4 b) a^{12} c) 2^6 d) x^4y^6 e) m^6 f) 2^{12}

4 a) $4^3 \times x^3$ b) $2^{2x+2} \times 3^{2x}$ c) $a^3 \times b^3$ d) vw^3 e) 9^2a^4 f) a^9b^9

5 a) $2^x \times 5^x$ b) $3^{2x} \times 7^{2x}$ c) 7^6 d) 5^{2x+2} e) 3^{12} f) $5^5 \times 7^5$

6 10^{120}

Activity 3

1 a) c^4 b) a^4 c) $p^1 = p$ d) 1 e) r f) b^8

2 a) 3^2 b) $\frac{1}{2^3}$ c) $3^0 = 1$ d) $10^0 = 1$ e) $10^0 = 1$ f) $12^1 = 12$

3 a) $\frac{1}{a^2}$ b) $\frac{1}{2^3}$

Activity 4

1 a) $a^{\frac{1}{3}}$ b) $2^{\frac{5}{2}}$ c) $\frac{1}{3}$ d) $2^{\frac{1}{3}}$ e) $\frac{1}{2^3}$ f) $\frac{1}{2^2}$

2 a) 2^1 b) $-\frac{1}{2^2}$ c) 3^2 d) $\frac{1}{3^4}$ e) $\frac{1}{5}$ f) $\frac{1}{7^2}$

3 a) 2^{16n-2} b) $\left(\frac{5}{6}\right)^2$ c) $\frac{3}{2}$ d) 7^2 e) 2^6 f) 2×5^2 g) x^3 h) c^2 i) a^2b^2

Activity 5

1 a) $x = 3$ b) $x = \frac{3}{2}$ c) $x = -1$ d) $x = 3$ e) $x = 0$ f) $x = -2$ g) $x = -2$ h) $x = \frac{4}{3}$ i) $x = 1$

2 a) $x = 6^3$ b) $x = \left(\frac{3}{2}\right)^{\frac{4}{3}}$ c) $x = \frac{2}{3}$ d) $x = 15$ e) $x = 9$ f) $x = 4$

3 $x = -3$ or $x = 1$

Activity 6

1 a) 128

2 a) 2^{12}; the t…

b) 2^{14}; the fo…

3 1 953 125 vo…

4 6 561 km

5 a) 125 cm³

6 a) 19 683 ce…

c) the twelfth…

d) 177 147 ce… generation…

Topic 3 Alg…

Starter activity

1 a) 5 tennis ba…

b) 3 footballs…

c) $32p$

e) $10m - n + 8$…

2 You can only a…

Activity 1

1 a) $8p + 2q$ d) $5l - 6k$ g) $7a - 7b$ i) $5xy + 3xz$

2 4 oranges + 3 b…

Activity 2

1 a) $2p + 2$ c) $5jk - 5j$ e) $9m - m^2$ g) $-9a + 3a^2$ i) $-pq^3 - 3p^3q$

2 32 pens + 12 pe…

Activity 3

1 a) $5k + 12$ d) $3y + 16$

2 a) $a^2 + 8a + 7$ c) $3f^2 + 5f + 2$ e) $5j^2 - 15jk +$… f) $6x^2 - 33xy +$… g) $-8e - 2$

Activity 4

1 $3(z + 3)$

3 $3(1 - 4e)$

5 $4r(3r + 2)$

7 $6e^2(4 + e^2)$

9 $6(1 - 2z - 3y)$

11 $b(7 + 7bc - b^2c^2)$

Activity 6

1 a) 128 b) 1 024
2 a) 2^{12}; the twelfth generation
b) 2^{14}; the fourteenth generation
3 1 953 125 votes
4 6 561 km
5 a) 125 cm^3 b) 7 cm
6 a) 19 683 cells b) 177 147 cells
c) the twelfth generation
d) 177 147 cells in the eleventh generation

Topic 3 Algebra

Starter activity

1 a) 5 tennis balls + 6 footballs
b) 3 footballs + 6 tennis balls
c) $32p$ d) $5a + 2p$
e) $10m - n + 8$ f) $3a^2 + 2a + 15$
2 You can only add or subtract like terms.

Activity 1

1 a) $8p + 2q$ b) $7x + 7y$ c) $-8r$
d) $5l - 6k$ e) $2m + 3n$ g) $-3d + 6e$
g) $7a - 7b$ h) 0
i) $5xy + 3xz$ j) $4jk - 2mn$
2 4 oranges + 3 bananas

Activity 2

1 a) $2p + 2$ b) $3x - 3y$
c) $5jk - 5j$ d) $3z^2 + 4z$
e) $9m - m^2$ f) $10r^2 + 6r$
g) $-9a + 3a^2$ h) $8xy + xy^2$
i) $-pq^3 - 3p^3q$ j) $16nm - 8n^2$
2 32 pens + 12 pencils

Activity 3

1 a) $5k + 12$ b) $6r + 4s$ c) $-2m - 13$
d) $3y + 16$ e) $2z$ f) $2ab^2$
2 a) $a^2 + 8a + 7$ b) $k^2 - 4$
c) $3f^2 + 5f + 2$ d) $10n^2 - 21n + 9$
e) $5j^2 - 15jk + 10k^2$
f) $6x^2 - 33xy + 15y^2$
g) $-8e - 2$ h) $-5b^2 + 6b - 25$

Activity 4

1 $3(z + 3)$ 2 $k(1 - j)$
3 $3(1 - 4e)$ 4 $x(y - z)$
5 $4r(3r + 2)$ 6 $5n(3n - 5)$
7 $6e^2(4 + e^2)$ 8 $qr(p + r)$
9 $6(1 - 2z - 3y)$ 10 $8x(8 - 7y - 6x)$
11 $b(7 + 7bc - b^2c^3)$ 12 $uv^2(1 + u^2 - v)$

Activity 5

1 $(p + q)(r + s)$ 2 $(j + k)(5 + ik)$
3 $(m + n)(a + b)$ 4 $(s - t)(r + 3)$
5 $(x - u)(6 + y)$ 6 $(b + 2)(a + 2)$
7 $(2 + y)(x - 3)$ 8 $(k - g)(f + k)$
9 $(3v - 4)(uv - 3)$ 10 $(z + 5)(a - 2)$
11 $(k + s)(k - 9)$ 12 $(2 - p)(3 - 2q)$

Activity 6

1 a) $(x + 5)(x + 1)$ b) $(x + 7)(x + 1)$
c) $(x + 4)(x + 2)$ d) $(5 + x)(4 + x)$
e) $(x - 4)(x - 4)$ f) $(x + 2)(x - 1)$
g) $(1 + x)(2 - x)$ h) $(5 - x)(6 + x)$
2 a) $2(k - 3)(k + 1)$ b) $(m + 1)(3m - 5)$
c) $(h + 3)(5h + 1)$ d) $(3x + 2)(3x - 1)$
e) $(2 + f)(3 - 4f)$ f) $(r + 2)(5r + 3)$
g) $(2n + 3)(3n + 1)$
h) $(s + 6)(2s - 5)$
3 a) $(a + b)(a + b) = (a + b)^2$
b) $(j - 4k)(j - 4k) = (j - 4k)^2$
c) $(r - 3s)^2$ d) $(5y - 2x)^2$

Activity 7

1 a) $(a - 5)(a + 5)$ b) $(p - q)(p + q)$
c) $(x - 3)(x - 3)$ d) $(3 - r)(3 + r)$
e) $(f - 4)(f + 4)$ f) $(10 - m)(10 + m)$
g) $(6 - y)(6 + y)$ h) $(x - 7)(x + 7)$
i) $5(x - 2)(x + 2)$ j) $(2 - 6p)(2 + 6p)$
k) $(2a - 3b)(2a + 3b)$
l) $(1 + i - j)(1 + i + j)$
2 a) 140 b) 260
c) 380 d) 11 000
3 $u - 2v$ 4 14

Activity 8

1 a) $x^2 + 2x + 1$ b) $k^2 - 4k + 4$
c) $9p^2 + 6p + 1$ d) $4s^2 - 12st + 9t^2$
e) $4a^2 - 16ab + 16b^2$
f) $15 + 10ab + a^2b^2$
2 a) $2i^2 + 2i + 13$ b) $2r^2 + 18$
c) $2m^2 + 4mn + 2n^2$
d) $2z^2 - 4z + 2$
e) $16k$ f) $12fg$

Activity 9

1 a) LCM: 200 b) LCM: p^2
c) LCM: $20a^2$ d) LCM: $60pq^2$
2 a) $\frac{2(6x - 1)}{y^2}$ b) $\frac{8m^2 + 15n^2}{6mn}$
c) $\frac{4p(q - 3p)}{3q^2}$ d) $\frac{3}{x^2}$

Activity 10

1 a) $\frac{7a}{12}$ b) $\frac{11y}{10}$ c) $1\frac{5k}{12}$

d) $\frac{1}{b}$ e) $\frac{2b}{c}$ f) $\frac{7x+10}{12}$

g) $\frac{11-x}{6}$ h) $\frac{12e-3}{5}$ i) $\frac{17m+1}{12}$

j) $3-3c^5$

2 a) $\frac{2u+3}{(u+2)(u+1)}$ b) $\frac{x-1}{(x+5)(x+3)}$

c) $\frac{4+8m}{(2-m)(2+m)}$ d) $\frac{10y+38}{(y-5)(3y+7)}$

e) $\frac{30-7b}{(4-b)(2-b)}$ f) $\frac{19e+20}{(2e+5)(e-3)}$

g) $\frac{10x-8}{(x-5)(2x-3)}$ h) $\frac{5r-s}{(r-s)(r+s)}$

i) $\frac{6p-p^2-2}{p-1}$ j) $\frac{3v^2+4v-5}{2v+2}$

3 a) $\frac{(y-3)}{(y+2)}$ b) $\frac{(e-2)}{(e+4)}$

4 a) $\frac{(x-3)}{(x+2)}$ b) $x=\frac{1}{2}$

Activity 11

1 $\frac{2x^2}{9}$ 2 $\frac{3y^2}{20}$ 3 x^3

4 $\frac{1}{ab}$ 5 $\frac{b^2}{2a}$ 6 $\frac{6y}{x}$

7 $\frac{yx}{14}$ 8 $\frac{5x^2}{6}$ 9 $\frac{3m}{10p}$

10 $\frac{4pm}{5}$

Activity 12

1 2 2 $\frac{2x}{5}$ 3 mf

4 $2f$ 5 $6a$ 6 $9q$

7 $\frac{15ax}{4}$ 8 $\frac{4x^2y^2}{3ab^2}$ 9 $\frac{2}{2}=1$

10 $11m$

Topic 4 Matrices

Starter activity

1

CDs	October	November	December
Rap	75	98	134
Classical	89	81	102

2 a) 2 by 2 b) 3 by 1

c) 2 by 3 d) 3 by 3

3 $\begin{bmatrix} 8 & 6 & 2 \\ 20 & 15 & 5 \\ 36 & 27 & 9 \end{bmatrix}$

Activity 1

1 a) 1 by 2 b) 2 by 2 c) 3 by 2

d) 2 by 3 e) 2 by 1 f) 3 by 3

2 Column matrix: E; Row matrix: A
Square matrices: B and F

3 Trailing diagonal of F: 0, 1, 0

4 a) Matrix B leading diagonal: 6, 7

b) Matrix F leading diagonal: 1, 1, 1

5 $x=2$
$y=\frac{1}{2}$

Activity 2

1 a) 1 by 2 b) 2 by 2 c) 3 by 2

d) 2 by 3 e) 2 by 1 f) 3 by 3

2 $A^T=\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $B^T=\begin{bmatrix} 6 & 7 \\ 8 & 7 \end{bmatrix}$

$C^T=\begin{bmatrix} 7 & 3 & 4 \\ 2 & 5 & 3 \end{bmatrix}$ $D^T=\begin{bmatrix} 6 & 2 \\ 4 & 9 \\ 3 & 1 \end{bmatrix}$

$E^T=[4 \quad 9]$ $F^T=\begin{bmatrix} 1 & 8 & 0 \\ 4 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

3

Order of matrix	A	B	C	D	E	F
Transpose	2 × 1	2 × 2	2 × 3	3 × 2	1 × 2	3 × 3
Matrix	1 × 2	2 × 2	3 × 2	2 × 3	2 × 1	3 × 3

4 a) $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 6 & 4 \\ 2 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 10 & -5 \\ -2 & 1 \end{bmatrix}$

Activity 3

1 a) $AB=\begin{bmatrix} 18 & 25 \\ 10 & 14 \end{bmatrix}$ b) $BA=\begin{bmatrix} 19 & 7 \\ 35 & 13 \end{bmatrix}$

c) 2 by 2 d) 2 by 2

2 a) $\begin{bmatrix} -6 & 4 \\ 9 & -6 \end{bmatrix}$ b) $NM=[-12]$

c) 2 by 2 d) 1 by 1

3 AB does not equal BA, and NM does not equal MN. The multiplication of two matrices is not commutative.

4 $\begin{bmatrix} 22 & 31 \\ 26 & 34 \\ 22 & 29 \end{bmatrix}$

Activity 4

1 a) $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 4 & 3 \\ 8 & 1 & 7 \\ 9 & 6 & 1 \end{bmatrix}$

2 a) $\begin{bmatrix} 3 & -2 \\ 7 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 3 & -2 \\ 7 & 4 \end{bmatrix}$

Activity 5

1 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 2 $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Activity 6

1 30 2 20 3 −22

4 −6

7 $\begin{bmatrix} -24 & 48 \\ -20 & 40 \end{bmatrix}$

10 $\begin{bmatrix} 9 & 22 \\ 6 & 10 \end{bmatrix}$

13 $\begin{bmatrix} 4 & 12 \\ 6 & 8 \end{bmatrix}$

16 $[12 \quad 9]$

19 $\begin{bmatrix} 26 & 68 & 10 \\ 40 & 64 & 14 \end{bmatrix}$

21 $\begin{bmatrix} 10 & 5 & 3 \\ 8 & 36 & -24 \\ 7 & -1 & 9 \end{bmatrix}$

Activity 7

1 1
4 −2
7 2
10 −2

Activity 8

1 $k=-3$

2 a) 23

c) 23

3 a) $k=3$ or -2

c) $k=-5$ or $k=2$

Activity 9

1 a) $\begin{bmatrix} 5 & -7 \\ 5 & 9 \end{bmatrix}$

d) $\begin{bmatrix} -9 & 1 \\ 5 & -3 \end{bmatrix}$

2 a) $\begin{bmatrix} -1 & -3 \\ -5 & -2 \end{bmatrix}$

3 a) −13

d) −13

4 a) $\begin{bmatrix} 3 & -2 \\ -1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 5 & -7 \\ 5 & 9 \end{bmatrix}$

g) $\begin{bmatrix} -9 & 1 \\ 5 & -3 \end{bmatrix}$

5 a) −2

d) 80

g) 22 h) 0

6 a) $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 9 & -5 \\ 7 & 5 \end{bmatrix}$

g) $\begin{bmatrix} -3 & -5 \\ -1 & -9 \end{bmatrix}$

4 −6

5 $\begin{bmatrix} -2 & -8 \\ -1 & -4 \end{bmatrix}$
6 $\begin{bmatrix} 8 & 12 \\ 80 & 120 \end{bmatrix}$

7 $\begin{bmatrix} -24 & 48 \\ -20 & 40 \end{bmatrix}$
8 $\begin{bmatrix} -6 & -6 \\ 8 & 18 \end{bmatrix}$
9 $\begin{bmatrix} 4 & 12 \\ 2 & 22 \end{bmatrix}$

10 $\begin{bmatrix} 9 & 22 \\ 6 & 10 \end{bmatrix}$
11 $\begin{bmatrix} 14 & 20 \\ 12 & 18 \end{bmatrix}$
12 $\begin{bmatrix} 14 & 4 \\ 6 & 14 \end{bmatrix}$

13 $\begin{bmatrix} 4 & 12 \\ 6 & 8 \end{bmatrix}$
14 $\begin{bmatrix} -1 & 0 \\ -28 & 82 \end{bmatrix}$
15 $\begin{bmatrix} -16 & -9 \\ -10 & -14 \end{bmatrix}$

16 [12 9]
17 $\begin{bmatrix} -2 & 2 \\ -26 & 8 \end{bmatrix}$
18 $\begin{bmatrix} 4 & 2 & 10 \\ 2 & 1 & 5 \\ 8 & 4 & 20 \end{bmatrix}$

19 $\begin{bmatrix} 26 & 68 & 10 \\ 40 & 64 & 14 \end{bmatrix}$
20 $\begin{bmatrix} 18 & 15 & 20 \\ 42 & 33 & 63 \\ 42 & 39 & 64 \end{bmatrix}$

21 $\begin{bmatrix} 10 & 5 & 3 \\ 8 & 36 & -24 \\ 7 & -1 & 9 \end{bmatrix}$

Activity 7

1 1　　2 0　　3 8
4 −2　　5 0　　6 3
7 2　　8 −22　　9 −14
10 −2　　11 −7　　12 −8

Activity 8

1 $k = -3$
2 a) 23　　b) 5
　c) 23　　d) 9
3 a) $k = 3$ or −2　　b) $k = 3$ or −2
　c) $k = -5$ or $k = 2$

Activity 9

1 a) $\begin{bmatrix} 5 & -7 \\ 5 & 9 \end{bmatrix}$　b) $\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$　c) $\begin{bmatrix} 3 & 2 \\ 5 & -4 \end{bmatrix}$
　d) $\begin{bmatrix} -9 & 1 \\ 5 & -3 \end{bmatrix}$　e) $\begin{bmatrix} 1 & 2 \\ 5 & 10 \end{bmatrix}$　f) $\begin{bmatrix} 0 & -2 \\ -4 & 6 \end{bmatrix}$
2 a) $\begin{bmatrix} -1 & -3 \\ -5 & -2 \end{bmatrix}$　b) $\begin{bmatrix} 5 & 3 \\ -2 & 8 \end{bmatrix}$　c) $\begin{bmatrix} 4 & -1 \\ -2 & 12 \end{bmatrix}$
3 a) −13　　b) 46　　c) 22
　d) −13　　e) 46　　f) 22
4 a) $\begin{bmatrix} 3 & -2 \\ -1 & 0 \end{bmatrix}$　b) $\begin{bmatrix} 2 & -3 \\ -3 & 1 \end{bmatrix}$　c) $\begin{bmatrix} 0 & -2 \\ -4 & 6 \end{bmatrix}$
　d) $\begin{bmatrix} 5 & -7 \\ 5 & 9 \end{bmatrix}$　e) $\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$　f) $\begin{bmatrix} 3 & 2 \\ 5 & -4 \end{bmatrix}$
　g) $\begin{bmatrix} -9 & 1 \\ 5 & -3 \end{bmatrix}$　h) $\begin{bmatrix} 1 & 2 \\ 5 & 10 \end{bmatrix}$
5 a) −2　　b) −7　　c) −8
　d) 80　　e) −19　　f) −22
　g) 22　　h) 0
6 a) $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$　b) $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$　c) $\begin{bmatrix} 6 & 4 \\ 2 & 0 \end{bmatrix}$
　d) $\begin{bmatrix} 9 & -5 \\ 7 & 5 \end{bmatrix}$　e) $\begin{bmatrix} -3 & -5 \\ -2 & 3 \end{bmatrix}$　f) $\begin{bmatrix} -4 & -5 \\ -2 & 3 \end{bmatrix}$
　g) $\begin{bmatrix} -3 & -5 \\ -1 & -9 \end{bmatrix}$　h) $\begin{bmatrix} 10 & -5 \\ -2 & 1 \end{bmatrix}$

Activity 10

1 a) $\begin{bmatrix} \frac{1}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{2}{13} \end{bmatrix}$　b) $\begin{bmatrix} \frac{5}{46} & \frac{3}{23} \\ \frac{-1}{46} & \frac{4}{23} \end{bmatrix}$
　c) $\begin{bmatrix} \frac{2}{23} & -\frac{1}{46} \\ -\frac{1}{23} & \frac{6}{23} \end{bmatrix}$　d) $\begin{bmatrix} 3 & 2 \\ -5 & -3 \end{bmatrix}$
　e) $\begin{bmatrix} -2 & \frac{5}{2} \\ -1 & \frac{-3}{2} \end{bmatrix}$　f) $\begin{bmatrix} -\frac{3}{4} & \frac{1}{4} \\ \frac{5}{4} & \frac{-3}{4} \end{bmatrix}$
2 a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$　b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
　c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$　d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
3 a) $\begin{bmatrix} \frac{1}{2} & \frac{-1}{3} \\ \frac{-1}{2} & \frac{2}{3} \end{bmatrix}$　b) $\begin{bmatrix} -13 & 2 \\ 7 & -1 \end{bmatrix}$
　c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$　d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
4 a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$　b) $\begin{bmatrix} 6 & 1 \\ 4 & 1 \end{bmatrix}$
　c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$　d) $\begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix}$
5 Multiplying a matrix by its inverse gives a unit matrix. Multiplying a matrix by a unit matrix leaves the matrix unchanged.

Activity 11

1 a) 0　b) 0　c) 0　d) 0
2 a) $\begin{bmatrix} \frac{-1}{13} & \frac{9}{52} \\ \frac{2}{13} & \frac{5}{52} \end{bmatrix}$　b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
3 a) $k = -13$　　b) $k = 16$
　c) $k = 12$　　d) $k = 8$ or $k = -4$

Activity 12

1 a) $x = 2$ and $y = -3$
　b) $x = 10$ and $y = -\frac{1}{5}$
2 a) $x = 1$ and $y = 1$
　b) $x = 2\frac{4}{5}$ and $y = \frac{3}{5}$
　c) $x = 2$ and $y = -2$
　d) $x = -1\frac{3}{7}$ and $y = 1$
　e) $x = 2$ and $y = -1$
　f) $x = -2$ and $y = 1$

Activity 13

1 $y = 3$
2 a) $x = \frac{4}{17}$ and $y = -1\frac{15}{17}$
　b) $m = 3$ and $n = \frac{1}{2}$
　c) $x = 10$ and $y = 2$
　d) $x = 5$ and $y = \frac{3}{13}$
　e) $m = 2$ and $n = -3$
　f) $x = 4$ and $y = -3$
　g) $x = 6$ and $y = 2$
　h) $v = 13$ and $z = -19\frac{1}{2}$

i) $a = 3$ and $b = -4$
j) $x = 6$ and $y = 9$
k) $x = 2$ and $y = -1$
l) $m = \frac{1}{3}$ and $n = -1$

3 Division by 0 is not allowed.

Activity 14

1 a) $4x + 0y = 360$ and $8x + y = 880$
b) $x = K90$ and $y = K160$

2 a) $4x + 2y = 29$ and $3x + 5y = 34$
b) $\begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 29 \\ 34 \end{bmatrix}$
c) $x = K5.50$ and $y = K7.50$

3 a)

x	$+y$	20 000
$0.12x$	$+0.04y$	$0.09 \times 20\,000$

b) 20 000 c) 1 800
d) $x = K12\,500$ and $y = K7\,500$

4 a) $C = \begin{bmatrix} 50 & 400 \\ 80 & 600 \\ 70 & 550 \end{bmatrix}$
b) $N = [28 \quad 30 \quad 34]$
c) $[6\,180 \quad 47\,900]$

Topic 5 Similarity and congruency

Starter activity

1

Dimensions	Length	Breadth	Breadth : length	Ratio: $\frac{\text{breadth}}{\text{length}}$
Large flag	7.5	5	7.5 : 5	$\frac{3}{2}$
Small flag	4.5	3	4.5 : 3	$\frac{3}{2}$

2 The ratios of corresponding sides is the same and all the angles are the same size.

3 a–c) Discussion questions

Activity 1

1 a) 3 cm
b) 300 km on the ground

2 6.6 cm
3 57 cm; 570 km

4 a) 9.05 km b) 70 cm on the map

5 a) 2 km b) 1 km c) 1.5 km

6 a) A: 1 : 2 000 000 B: 1 : 4 000 000
C: 1 : 10 000 000 D: 1 : 50 000 000
b) A: $\frac{1}{2,000,000}$ B: $\frac{1}{4,000,000}$
C: $\frac{1}{10,000,000}$ D: $\frac{1}{50,000,000}$

Activity 2

1 a) 800 m b) 400 m
c) 8 cm^2 d) 320 000 m^2

2 Scale: 1 cm : 25 cm (1 cm to 0.25 m)

	Description of feature	On the real train	On the model
a)	Length of engine	16×0.25 = 4 m	16 cm
b)	Length of train	20 m	$20 \div 0.25$ = 80 cm
c)	Height of train	12×0.25 = 3 m	12 cm
d)	Diameter of wheels	1.25 m	$1.25 \div 0.25$ = 5 cm
e)	Distance between wheels	6×0.25 = 1.5 m	6 cm

3 a) 1 : 200 b) 0.45 cm

4 1 : 100

5 a) $k = \frac{1}{2} \times 10 = 5$ b) $k = \frac{4}{12} \times 9 = 3$
c) $k = \frac{6}{4} \times 12 = 18$

Activity 3

1 a) $\frac{1}{10,000}$ b) 120 cm^2 c) 120 m^2

2 a) 120 m^2 b) $4 \times (8 \times 15)$
c) 480 m^2

3 a) $\frac{1}{3}$ b) 243 m^2 c) $\frac{1}{9}$
d) 27 m^2

Activity 4

1 a) 1 : 10 b) Length: 23 mm
Height: 7.5 mm
c) 1 : 1 000
d) 1 897.5 cm^3 e) 1.8975 cm^3
f) 1 897.5 cm^3

2 a) 2 : 3 b) 4 : 9 c) 184 cm^3

3 a) 1 200 cm or 1.2 m
b) 660 cm
c) 335 000 000 cm^3 or ml or 335 000 ℓ

4 a) 6 cm^2 b) 60 cm^3
c) Width: $1.5 \times 4 = 6$ cm
Length: $1.5 \times 10 = 15$ cm
Side length: $1.5 \times 3.6 = 5.4$ cm
d) 8 : 27 e) 202.5 cm^3

Activity 5

1 In △ABE and △DCE:
AB = DC (given)
$A\hat{B}C = D\hat{C}E$ (given)
$A\hat{E}B = D\hat{E}C$ (vertically opposite)
Therefore, △ABE ≡ △DCE (AAS)

2 a) right angle, hypotenuse, side (RHS)
b) side, angle, side (SAS)
c) side, side, si
d) angle, angle
(AAS)

3 Discussion ques

Topic 6 Travel

Starter activity

1 S × T = D, $\frac{D}{T}$ = S

2 a) Speed = $\frac{800}{32}$
b) Speed = $\frac{25}{1}$ m

3 The graph show
of, say 900 km,
many kilometre
hours, will be.

Activity 1

1 a) 4 km/h
c) 3 km

2 a) 150 km

3 a) 1.77 m/s
b) 6.372 ≈ 6.37

4 a) 9.89 m/s b)

Activity 2

1 scalar 2
4 scalar 5
7 vector

Activity 3

1 a) i) 40 km
iii) 60 km
b) i) $\frac{1}{4}$ h or 1
ii) $1\frac{1}{2}$ h
iv) 2 h
c) i) 20 km/h
iii) 20 km/h
d) The cyclist s
he stayed in
e) No, he stopp
he started hi

2 a) He ran 14 km
point and 14
starting point
b) He ran for 6 k
ran 8 km befo
turned back a
and then ran
c) He stopped t
30 min. and

d) 320 000 m^2
…25 cm (1 cm to 0.25 m)

…the real train	On the model
16 × 0.25 = 4 m	16 cm
20 m	20 ÷ 0.25 = 80 cm
12 × 0.25 = 3 m	12 cm
1.25 m	1.25 ÷ 0.25 = 5 cm
6 × 0.25 = 1.5 m	6 cm

b) 0.45 cm

…= 5 b) $k = \frac{4}{12} \times 9 = 3$
…12 = 18

b) 120 cm^2 c) 120 m^2
b) 4 × (8 × 15)

b) 243 m^2 c) $\frac{1}{9}$

b) Length: 23 mm
Height: 7.5 mm
… e) 1.8975 cm^3
b) 4 : 9 c) 184 cm^3
…cm or 1.2 m
…000 cm^3 or ml or 335 000 ℓ
b) 60 cm^3
…: 1.5 × 4 = 6 cm
…: 1.5 × 10 = 15 cm
…ngth: 1.5 × 3.6 = 5.4 cm
e) 202.5 cm^3

…and ΔDCE:
(given)
…CE (given)
…C (vertically opposite)
…, ΔABE ≡ ΔDCE (AAS)
…angle, *hypotenuse, side (RHS)*
…angle, side (SAS)

c) side, side, side (SSS)
d) angle, angle, corresponding side (AAS)
3 Discussion question

Topic 6 Travel graphs

Starter activity

1 S × T = D, $\frac{D}{T}$ = S or $\frac{D}{S}$ = T
2 a) Speed = $\frac{800}{32}$ m/s = 25 m/s
b) Speed = $\frac{25}{1}$ m/s = $\frac{25 \times 3\,600}{1\,000}$ = 90 km/h
3 The graph shows you how long a flight of, say 900 km, will take and how many kilometres a flight of say, seven hours, will be.

Activity 1

1 a) 4 km/h b) $\frac{1}{4}$ h or 15 min.
c) 3 km
2 a) 150 km b) $\frac{1}{2}$ h or 30 min.
3 a) 1.77 m/s
b) 6.372 ≈ 6.37 km/h
4 a) 9.89 m/s b) 35.604 ≈ 35.6 km/h

Activity 2

1 scalar 2 vector 3 vector
4 scalar 5 scalar 6 vector
7 vector

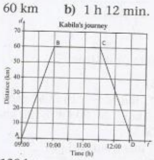
Activity 3

1 a) i) 40 km ii) 60 km
iii) 60 km iv) 70 km
b) i) $\frac{1}{4}$ h or 15 min.
ii) $1\frac{1}{2}$ h iii) $\frac{1}{2}$ h or 30 min.
iv) 2 h
c) i) 20 km/h ii) 60 km/h
iii) 20 km/h iv) 10 km/h
d) The cyclist stopped for 30 minutes – he stayed in the same place.
e) No, he stopped 70 km from where he started his journey.
2 a) He ran 14 km from the starting point and 14 km back to the starting point; a total of 28 km.
b) He ran for 6 km, stopped and then ran 8 km before stopping, he then turned back and ran 8 km, stopped and then ran the last 6 km.
c) He stopped three times (15 min., 30 min. and 15 min.).

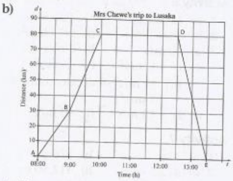
d) i) 8 km/h ii) 10.67 km/h
iii) 8 km/h iv) 12 km/h
e) i) 7 km/h ii) 9.33 km/h
3 a) 09:45 b) 14 km
c) He and Mate visited for one hour.
d) 8 km e) 15 min.
f) i) 20 km/h ii) 2.29 km/h
iii) 16 km/h iv) 24 km/h

Activity 4

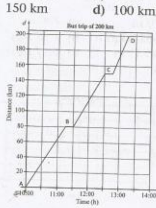
1 a) 60 km b) 1 h 12 min.
c)



d) 120 km
2 a) 40 km/h
b)

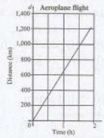


c) She arrived home at 13:30.
3 a) 64 km/h b) 11:30
c) 150 km d) 100 km/h
e)



d) She arrived at 13:15.

4 a) i) 1 211 km S
ii) 1 211 km N
b) 660 km/h N
c)



d) Vector quantities

Activity 5

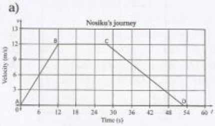
1 a) 27.78 m/s b) 3.52 m/s^2
2 a) 7.5 km b) 20 km/h; 50 km/h
c) 100 km
3 a) 78 m/s (rounded off)
b) 39 s c) 78 s

Activity 6

1 a) 12.42 m/s b) 2.22 s
2 a) 3 s b) 9.33 m/s^2
3 a) 10 m/s b) 2 m/s^2
c) i) 1 m/s^2 ii) $\frac{1}{2}$ m/s^2
iii) –1 m/s^2 or deceleration of 1 m/s^2
d) i) 15 m/s ii) 10 m/s

Activity 7

1 a) 126 b) 126 m
2 a) 412.5 b) 412.50 m
3 a) 75 km/h b) 10 seconds
c) i) 15 m/s^2 ii) 7.5 m/s^2
iii) –45 m/s^2
d) 570 m e) 780 m
4 a) 100.08 km/h b) 7.62 m/s^2
c) 50.735 m d) 6.47 m/s^2
e) 59.77 m
5 a)



b) 1 m/s^2 c) 0.48 m/s^2
d) 414 m

Topic 7 Social and commercial arithmetic

Starter activity

1 Answers depend on the information learners have found.
2 Answers will differ.
3 Answers include the internet, newspapers, radio and television broadcasts and banks.
4 Discussion question

Activity 1

1 a) $10 186 b) $1 527.90
2 a) $472 851 ≈ $472 85
b) 1 000 c) K8.73
d) K276 830 e) 746
f) K5 643 506.40 ≈ K5 643.500
3 a) K8.98 b) K3.33
c) K3.70 d) K0.47
4 a) K20 000 b) K28 000
c) K8 000 d) 40%
5 a) Companies issue (sell) shares to raise capital for expansion.
b) 55 000 shares c) K1 265 000
d) 143.5% e) K5 600 000

Activity 2

1 a) K0.78 b) K11 700
2 a) K1.0675 b) K11.40
3 a) K14 625.00 b) K1 475.00
c) 10.09% d) K204.75

Activity 3

1 a) 30 000 units b) K9 000 000
c) K45 000 000 d) K5 000
e) K1 500
2 a) K500 b) K20 000
3 K630.00
4 a) Year 1: K677.25 Year 2: K728.04
Year 3: K782.65 Year 4: K841.35
b) Year 2: K677.25 Year 3: K728.04
Year 4: K782.65
c) Year 3: K677.25 Year 4: K728.04
d) Year 4: K50.79 e) K98.04 ≈ K98.00
5 a) K1 485 000 b) K3 616 250
6 Commercial Bank interest: 240 000 000
Amount owed: K2 240 000 000
Bank of Zambia interest: 40 000 000
Amount owed: K1 040 000 000
Total owed: K3 280 000 000

Topic 8 Bear

Starter activity

Solwezi is north
northeast of Kasem

Activity 1

1 Draw angles.
2 a) 80° b) 3
3 a) 50° b) 9
e) 138° f) 1

Activity 2

1

Village
My town
Our town
Big town
Your town

2 a, b)

3

Name
Musa
Kasuba
Monde
Katebe
Milupi
Teza
Chanda

4

Aeroplane
1
2
3
4
5

Activity 3

1 a) P to Q: 75°
b) P to Q: 100°

Topic 8 Bearings

Starter activity

Solwezi is northwest of Kitwe and north-northeast of Kasempa.

Activity 1

1 Draw angles.

2 a) 80° b) 100° c) 40° d) 180°

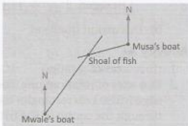
3 a) 50° b) 90° c) 283° d) 180° e) 138° f) 110° g) 297° h) 73°

Activity 2

1

Village	Bearing
My town	051°
Our town	113°
Big town	313°
Your town	225°

2 a, b)



3

Name	Calculation	Bearing
Musa	25°	025°
Kasuba	25° + 25°	050°
Monde	90°	090°
Katebe	180° – 62°	118°
Milupi	180° + 38°	218°
Teza	270° – 22°	218°
Chanda	360° – 27°	333°

4

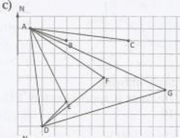
Aeroplane	Compass direction	Bearing
1	N 34° W	326°
2	N 52° E	052°
3	SE	135°
4	S 81° W	261°
5	N 74° W	286°

Activity 3

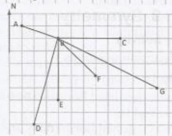
1 a) P to Q: 75° and Q to P: 255°
b) P to Q: 100° and Q to P: 280°

2 Allow two degrees measurement error.

a, c)

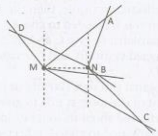


b)



a) A of B: 108°; of C: 097°; of D: 173°; of E: 153°; of F: 124°; of G: 114°
b) B of A: 288°; of C: 090°; of D: 196°: of E: 180°; of F: 135°; of G: 120°
c) D of E: 045°; of F: 051°; of C: 045°; of G: 073°

3



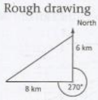
Activity 4

1 a) Scale drawing
b) B from C: 304° c) C from A: 040°
d) Approximately 93 km

2 Scale drawing
a) approximately 13.3 km
b) approximately 20°

3 a) Camp B is 3 km north of camp A.
b) Camp A is 6 km west of camp C.
c) approximately 37°
d) approximately 6.3 cm
e) 6.3 km

4 a) Rough drawing



b) Scale drawing
c) Distance is approximately 10 km.
d) 10

Topic 9 Symmetry

Starter activity

1, 2 Discussion questions

Activity 1

1 A was translated horizontally to create B, C and D.

2 A was reflected horizontally and translated to give B; translated to give C, and reflected horizontally and translated to give D.
Or, A was rotated through 180° to give B; translated to give C; rotated through 180°, and then translated to give D.

3 A was reflected vertically, then the reflection was translated to give B; A was translated to give C and then A was reflected vertically and translated to give D.

4 A was rotated through 180° about the midpoint of the longest side to give B.

5 A was reflected about its longest side to give B; translated to give C; reflected about the longest side to give D; reflected and translated downward to give E; translated downward to give F; and so on.

Activity 2

1 a) reflectional and rotational symmetry of order 3
b) reflectional symmetry with a vertical line through the centre and rotational symmetry of order 2
c) a horizontal and a vertical line of symmetry and rotational symmetry of order 2
d) reflectional symmetry about a vertical line through the middle of the shape
e) rotational symmetry of order 6
f) six lines of symmetry and rotational symmetry of order 6

2 a) five lines of symmetry
b) 5

3 a) Diagram A can be reflected or rotated through 120° to create diagram B.
b) Diagram C be reflected through the centre of the diagram or rotated through an angle of 120° to create diagram D.

4 a) 2 b) 2 c) 2

5 No lines of symmetry

6 a, c)

	Order of rotational symmetry	Number of lines of symmetry
Triangle	3	3
Pentagon	5	5
Hexagon	6	6

b) Discussion question

Activity 3

1 three axes of rotation

2 four axes of rotation: one along each edge where two rectangles meet and one through the centre of the triangular face

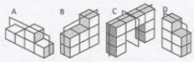
3 one axis of rotation

Activity 4

1 rectangular prism: 3
triangular prism: 4
square-based pyramid: 2

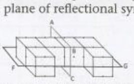
2 a) six b) two

3



4 a) There are three planes of symmetry.
b) There are five planes of symmetry.

5 a) A plane of reflectional symmetry



b) An axis of rotational symmetry

6 a) One axis of rotational symmetry
b) Perpendicular to the base through the vertex
c) An infinite number of planes of reflection symmetry

Topic 10 Co
calculator

Starter activity

1 a) 128
d) $\frac{22}{7}$ = 3.14

2a, b) Discussion

Activity 1

1 a) 326.34408
c) 26.0742
e) $\frac{49}{60}$

2 a) 7.4444... =
b) 1.45

3 a) $\frac{9}{25}$

Activity 2

1 a) 256
d) 48 841

2 a) 64
d) 3 375

3 a) 1 024
d) 3 125

4 a) 9 604

5 a) 549.78 cm
c) 169.65 cm

6 a) 225 cm^2

7 a) $\frac{1}{25}$

8 a) 0.04

Activity 3

1 a) 21
d) 3.87

2 a) 9
d) 15

3 a) 2
d) 2.1

4 a) 45

5 a) −1.2
c) 19.23

Activity 4

1 a) 3.9375 × 10
c) 3.083 × 10^5

2 1.04

3 a) 1.07 × 10^{-1}

Activity 5

1 $2 \times 3^2 \times 5^3$

3 $2^3 \times 3^4 \times 5$

Topic 10 Computer and calculator

Starter activity

1 a) 128 b) 21 c) 27
d) $\frac{22}{7} = 3.142857143$

2a, b) Discussion questions

Activity 1

1 a) 326.344086 b) 0.1027100917
c) 26.0742 d) $\frac{4}{5}$
e) $\frac{49}{60}$ f) $7\frac{1}{5}$

2 a) $7.4444... = 7.\dot{4}$
b) 1.45 c) 3.84

3 a) $\frac{9}{25}$ b) $\frac{1}{8}$ c) $\frac{3}{8}$

Activity 2

1 a) 256 b) 729 c) 33 124
d) 48 841 e) 250 000

2 a) 64 b) 343 c) 1 000
d) 3 375 e) 91.125

3 a) 1 024 b) 32 768 c) 729
d) 3 125 e) 19.4481

4 a) 9 604 b) 1 025 c) 468

5 a) 549.78 cm^3 b) 1 178.10 cm^3
c) 169.65 cm^3

6 a) 225 cm^2 b) 25 cm^2 c) 49 cm^2

7 a) $\frac{1}{25}$ b) $\frac{1}{4}$ c) $\frac{1}{2}$

8 a) 0.04 b) 0.25 c) 0.5

Activity 3

1 a) 21 b) 38 c) 5.66
d) 3.87 e) 50

2 a) 9 b) 12 c) 3
d) 15 e) 3.56

3 a) 2 b) 6.45 c) 3
d) 2.1 e) 4

4 a) 45 b) 20 c) 0.37

5 a) −1.2 b) 1.02
c) 19.23 d) 1.45

Activity 4

1 a) 3.9375×10^{13} b) 4.4×10^{-13}
c) 3.083×10^5 d) 1.53×10^{-3}

2 1.04

3 a) 1.07×10^{-1} b) 9.345×10^3

Activity 5

1 $2 \times 3^2 \times 5^3$
2 $2 \times 3 \times 7 \times 13$
3 $2^3 \times 3^4 \times 5$
4 $2^2 \times 3^3 \times 7$
5 $3^2 \times 5^2 \times 7^2$
6 $2^3 \times 3 \times 5$
7 $2^2 \times 3^3$
8 $2^3 \times 3^2 \times 7$

Activity 7

1 Monitor
2 Tower case
3 Mouse
4 Speakers
5 Printer
6 Keyboard
7 Camera

Activity 8

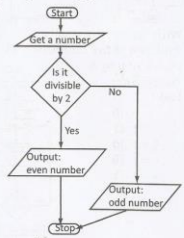
1 Wake up; make your bed; take a bath; clean your teeth; get dressed; eat breakfast; drink tea; wash the dishes; walk to school.

2 Find a mark; divide mark by 50; multiply answer by 100; write down the percentage.

Activity 10

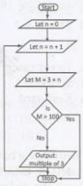
1 a) 1964 → ÷ 4 → yes → 1964 ÷ 100 → no → leap year
b) 1970 → ÷ 4 → no → not a leap year
c) 2014 → ÷ 4 → no → not a leap year
d) 2020 → ÷ 4 → yes → 2020 ÷ 100 → no → leap year
e) 1600 → ÷ 4 → yes → 1600 ÷ 100 → yes → 1600 ÷ 400 → yes → leap year
f) 1700 → ÷ 4 → yes → 1700 ÷ 100 → yes → leap year
g) 1800 → ÷ 4 → yes → 1800 ÷ 100 → yes → leap year
h) 1900 → ÷ 4 → yes → 1900 ÷ 100 → yes → leap year

2



3 81 → odd; 82 → even; 83 → odd; 84 → even; and so on, and 99 → odd

4



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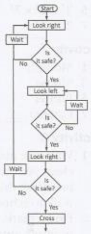

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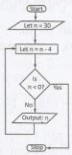
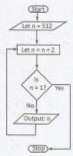
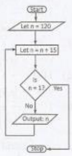


Activity 11

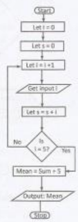
1 a)



b)



2 1 Write inputs: five numbers.
  2 Find sum of five numbers.
  3 Divide sum by 5.
  4 Print answer.
  5 End.

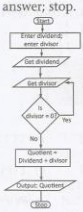


3 1 Write the inputs: pi; radius
  2 Get *r*
  3 Get pi
  4 Let A = pi × *r* × *r*
  5 Print A
  6 End

4 Enter *r*; enter *h*; enter *π*; let V = π × *r* × *r* × *h*; print V; stop.

5 Enter *y*; enter *x*; calculate x^y; print answer; stop.

6

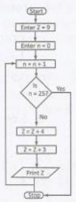


7 Enter *r*; enter *h*; enter *π*; let S = 2 × π × *r* × (*r* + *h*); print S; stop.

8 Let hypotenuse be *c*; get *a*; get *b*; let *c* = sqrt (*a* × *a* + *b* × *b*); print *c*; *end*.

Activity 12

1



2 You may run the program using a calculator (as in question 3).

3 The answer tends to the value 4.

4 The answer tends to the value 4.

5 a)
```
10 x = 30
20 FOR N = 1 TO 25
30 x = x/5
40 x = x + 7
50 PRINT X
60 NEXT N
70 END
RUN
```
b) Answers may differ.
c) The answer tends to 8.75.
d,e) Do and then repeat the activity.
f) The answer tends to 7.875.

Revision

Topic 1: Revision exe

1 a) 4
c) 2
e) 2
g) 4

2 a) False
c) True
e) True

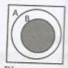
3 a) {a, c, e, f, g, h,
b) {a, b, c, d, i, k}
c) {a, c}
d) {a, b, c, d, e, f, g,
e) {b, d, i, j, k, l}
f) {b, d, e, f, g, h, i,
g) {j, l, m}
h) {b, d, i, k}

4 a) (A ∪ B)' ∪ (A ∩ B
b) [(A ∩ B) ∪ (B ∩ C

5 a) 28
c) 5
e) 6

6 a) 11
b) 2
c) 24

7 Diagram 1
a) A ∩ B



c) B'

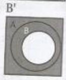
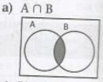
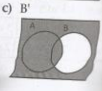


Diagram 2
a) A ∩ B



c) B'



pi; radius

print V; stop.
calculate x^y; print

7 Enter r; enter h; enter π; let S = 2 × π × r × (r + h); print S; stop.
8 Let hypotenuse be c; get a; get b; let c = sqrt (a × a + b × b); print c; *end*.

2 You may run the program using a calculator (as in question 3).
3 The answer tends to the value 4.
4 The answer tends to the value 4.

Revision exercises: answers

Topic 1: Revision exercises

1 a) 4 b) 3
c) 2 d) 2
e) 2 f) 3
g) 4 h) 1

2 a) False b) False
c) True d) True
e) True f) True

3 a) {a, c, e, f, g, h, n}
b) {a, b, c, d, i, k}
c) {a, c}
d) {a, b, c, d, e, f, g, h, i, k, n}
e) {b, d, i, j, k, l}
f) {b, d, e, f, g, h, i, j, k, l, n}
g) {j, l, m}
h) {b, d, i, k}

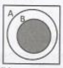
4 a) (A ∪ B)' ∪ (A ∩ B)
b) [(A ∩ B) ∪ (B ∩ C) ∪ (A ∩ C)]'

5 a) 28 b) 28
c) 5 d) 16
e) 6 f) 13

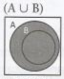
6 a) 11
b) 2
c) 24

7 Diagram 1

a) A ∩ B



b) (A ∪ B)

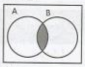


c) B'

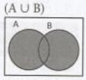
d) (A ∩ B)'

Diagram 2

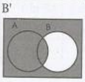
a) A ∩ B



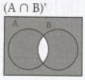
b) (A ∪ B)



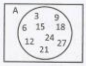
c) B'



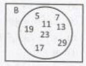
d) (A ∩ B)'



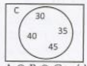
8 a)



b)



c)



A ∩ B ∩ C = { }

9 {12, 24, 36, 48}
{12, 24, 36, 48}
A = B

Topic 2: Revision exercises

1 a) $\frac{4}{a}$ b) $3r^3$
c) $\frac{1}{b^3}$

2 a) p^2 b) a^4b^2
c) $\frac{1}{y^4}$ d) y
e) a^4 f) t^6
g) $(xy)^{-13}$ h) $3ab$
i) $\frac{5a}{2b}$

3 a) $\sqrt{3}$ b) $\sqrt[3]{5^2}$
c) $\sqrt[3]{8^4}$ d) $\sqrt[4]{6^3}$
e) $\sqrt{\frac{1}{2}}$ f) $\sqrt{\frac{1}{3^3}}$

4 a) $a^{\frac{2}{3}}$ b) $a^{\frac{4}{3}}$
c) $a^{\frac{3}{5}}$ d) $a^{\frac{3}{4}}$
e) $a^{-\frac{1}{2}}$ f) $a^{-\frac{5}{2}}$

5 a) 2 b) 3
c) 4 d) 4
e) 4 f) 1

6 a) $x = 1$ b) $x = 2$
c) $x = 1$ d) $x = 3$
e) $x = 0$ f) $x = 1$

Topic 3: Revision exercises

1 a) $12p + 3q$
   b) $6a$
2 a) $4p + p^2$
   b) $2a^2b - ab^2$
   c) $x^2 - 2xy + y^2$
   d) $n - 8$
   e) $3a^2 + 15a + 2ab + 10$
   f) $9y^2 - 6y + 1$
3 a) $4(y - 3)$
   b) $a(a - b)$
   c) $4ab - 2b$
   d) $(a - 4)(a + 4)$
4 a) $(p + 5)(p + 2)$
   b) $(b - 4)(b - 1)$
   c) $(3 - y)(4 + y)$
   d) $(3x + 4)(x - 4)$
   e) $(3 + x)(5 - 3x)$
   f) $2(2x + 5)(x + 1)$
5 a) $\frac{1}{b}$
   b) 0
   c) $\frac{2(3m - 7)}{(m - 3)(m - 2)}$
   d) $\frac{(y + 4)}{(y + 1)}$

Topic 4: Revision exercises

1 a) 1 by 2   b) 1 by 3
   c) 2 by 1   d) 3 by 1
   e) 2 by 3   f) 3 by 3
2 a) $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$ b) $\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$
   c) [2 5]   d) [2 4 7]
   e) $\begin{bmatrix} 2 & 9 \\ 5 & 2 \\ 5 & 3 \end{bmatrix}$ f) $\begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 7 \\ 2 & 5 & 2 \end{bmatrix}$
3 a) $y = 5$
   b) $x = 6$
4 a) $\begin{bmatrix} 26 \\ 36 \\ 30 \end{bmatrix}$ b) $\begin{bmatrix} 45 & 37 & 49 \\ 84 & 43 & 61 \\ 92 & 55 & 92 \end{bmatrix}$
5 a) $\begin{bmatrix} 218 & 36 \\ 180 & 104 \end{bmatrix}$ b) $\begin{bmatrix} 218 & 216 \\ 30 & 104 \end{bmatrix}$
   c) $\begin{bmatrix} 169 & 132 \\ 110 & 136 \end{bmatrix}$ d) $\begin{bmatrix} 316 & 120 \\ 100 & 136 \end{bmatrix}$
6 Division by 0 is not allowed.

7 a) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -1 \\ -4 & 5 \end{bmatrix}$
   c) $\begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ -2 & 1 \end{bmatrix}$ d) $\begin{bmatrix} \frac{3}{-4} & \frac{1}{4} \\ \frac{5}{-4} & \frac{3}{4} \end{bmatrix}$
8 a) $x = 2$ or -2
   b) $x = \pm\sqrt{10}$
   c) $x = 7$ or -5
   d) $x = 6$ or 2
9 a) $x = 2$ and $y = 2$
   b) $x = 3$ and $y = \frac{1}{2}$
   c) $x = 2$ and $y = -10$
10 a) [22 12 80]   b) $\begin{bmatrix} 20 \\ 50 \\ 30 \end{bmatrix}$
   c) Ngwee 3 440
11 a) K297.50
   b) $x = 4\,000$ and $y = 297.50$
   c) $x =$ K1 500; $y =$ K2 500
12 $x =$ K666.67 and $y =$ K3 333.33
13 $x =$ K2 700; $y =$ K1 166.67

Topic 5: Revision exercises

1 a) 1 200 km
   b) 2 320 km
   c) 3.09 cm
2 A: 3 : 1; B: 2 : 1; C: 3 : 1; D: 2 : 1
   Pairs of similar cones: A and C, and B and D
3 a) 7 cm   b) 70 cm   c) 11 cm
4 a) 1 : 5   b) 1 : 5
5 7.56 m

Topic 6: Revision exercises

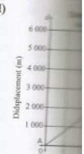
1

	Distance	Speed	Time
a)	28 m	7 m/s	4 s
b)	180 km	120 km/h	1.5 h
c)	45 km	15 km/h	3 h
d)	49 m	2 m/s	24.5 s
e)	280 km	70 km	4 h

2

	Initial velocity	Final velocity	Acceleration	Time
a)	28 m/s	44 m/s	4 m/s	4 s
b)	0 m/s	27.8 m/s	3.5 m/s	7.9 s
c)	0 m/s	45 m/s	15 m/min.	3 min
d)	49 m/s	0 m/s	–7 m/s	7 s

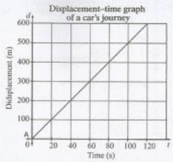
3 a)


   b) 5 m/s
   c) 18 km/h
4 a) 90 000 m
   b) 4.62 m/s
   c) 5.413 h
   d) 16.63 km/h
5 a) 11°
   b) 5 036 km
   c) 774.77 =
   d)


   e) 5 036 km
6 a) 10 m/s
   b) 5.5 s
   c) 2.5 m/s^2
   d) 0 m/s^2
   e) 1.33 m/s^2
   f) 98 m

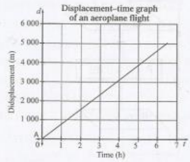
Topic 7: Revision ex

1 a) K8 170.20
   b) K1 050
   c) €9 000
2 a) K8 710
   Liya paid K8 7
   b) K485.75
   c) K5 678.25
3 a) K136 000 000
   b) 35.71%
4 a) K11 250 000

3 a) 

b) 5 m/s
c) 18 km/h

4 a) 90 000 m; 19 487 s
b) 4.62 m/s
c) 5.413 h
d) 16.63 km/h

5 a) 11°
b) 5 036 km E 11°
c) 774.77 ≈ 775 km/h E 11°
d) 

e) 5 036 km SW 11°

6 a) 10 m/s
b) 5.5 s
c) 2.5 m/s^2
d) 0 m/s^2
e) 1.33 m/s^2
f) 98 m

Topic 7: Revision exercises

1 a) K8 170.20
b) K1 050
c) €9 000

2 a) K8 710
Liya paid K8 710 for the shares.
b) K485.75
c) K5 678.25

3 a) K136 000 000
b) 35.71%

4 a) K11 250 000
b) K9 281 250

5 a) R34.38
b) R41.25
c) R1 093.20

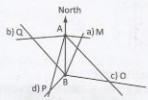
Topic 8: Revision exercises

1 a) M from Q: 211°; P from M: 120°
b) M from Q: 291°; P from M: 180°
c) M from Q: 323°; P from M: 264°
d) M from Q: 236°; P from M: 319°

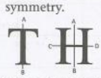
2 a) N 30° E: 030°
b) East: 090°
c) S 34° E: 146°
d) W: 270°
e) NW: 315°
f) S 55° W: 235°

3 025°

4 232°

5 

Topic 9: Revision exercises

1. a) Only T and H have lines of symmetry.


b) T has one line of symmetry.
H has two lines of symmetry.
S and A have no lines of symmetry.
c) Only H has a centre of rotation in its centre.
d) Order of rotation for T and A: 0; order of rotation for H and S: 2

2

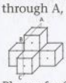
Shape	Order of rotational symmetry	Number of lines of symmetry
Triangle	3	3
Pentagon	5	5
Hexagon	6	6
Octagon	8	8

3 a) 2
b) 2

4 a) Two lines of reflectional symmetry

b) No lines of reflectional symmetry.

5 a) Plane of reflectional symmetry through A, B and C.



b) Plane of reflectional symmetry through A, B and C; one through D, E and F is is perpendicular to the plane through A, B and C.

6 a, b)Discussion questions

7

	Object	Number of rotational axes	Number of planes of symmetry
a)	Tetrahedron	7	6
b)	Pentagonal prism	6	6
c)	Square pyramid	1	2
d)	Cylinder	1	infinite

Topic 10: Revision exercises

1 a) –0.92
b) 2.533
c) –5.371
d) 1.272

2 a) $\frac{12}{25}$
b) $2\frac{2}{5}$
c) $-\frac{31.003}{1.000}$
d) $\frac{14}{25}$

3 a) 256
b) 3 136
c) 13 689
d) 289
e) 2 197
f) 15 625
g) 6 561
h) 293 162.506

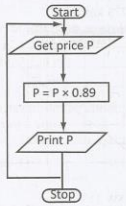
4 a) 4
b) 3.576
c) 5
d) 2.118

5 a) 8.6×10^{-6}
b) 1.29×10^{14}
c) 2.079×10^{-4}
d) 1.5×10^{8} km
e) 3.8×10^{-4} mm

6 Discussion question

7 Programs that make it possible for the different parts of the computer to work.

8 Get the price → multiply the price by 0.89 (89%) → print new price



9 Choose x_1 → substitute in equation → calculate y_1 → repeat for x_2 and y_2 → calculate $\frac{(y_2 - y_1)}{(x_2 - x_1)}$ → end.

10 Let $x = 0$ → substitute in equation → calculate y_0 → let $y = 0$ → substitute in equation → calculate x_0 → print $(0; y_0)$ → print $(x_0; 0)$ → end.

11 Discussion questions

make it possible for the
of the computer to work.
→ multiply the price by
print new price

→ substitute in equation
→ repeat for x_2 and y_2
→ end.
substitute in equation →
→ let $y = 0$ → substitute in
calculate x_0 → print (0; y_0)
→ end.
questions

Study and exam skills

Every day counts

As you know, football players practise their skills by running, kicking the ball and defending. They do not stand on the sideline, watch their coach or another player demonstrate moves, think that they understand what is going on and then go home without kicking a ball. The best football players work on their fitness and they practise ball skills every day.

To be successful in Mathematics you have to practise your skills. You cannot stand on the sideline. You cannot just watch your teacher explain something, think that you understand, close your books and walk away. Ask questions when you do not understand something. Listen to explanations. Apply new information. Use the answers that are given in this *Learner's Book* to check whether you are on track or whether you need to ask for help. This will help you to improve your skills. Every day in class makes a difference.

Make sure tests and exams count

In a football match, players have to handle each situation as it arises. They need to read the game and react appropriately. In a test or an exam, you need to read questions very carefully and make sure you understand what to do before you start answering a question. Read questions again if necessary. If you do not know how to answer a question, move to the next question. Return to the questions you have left out if you have time left after the last question.

When writing tests and exams check the following:

- Do you have to show your calculations? Marks are allocated for showing calculations and not only for the answers.
- What is the mark allocation for a question? If a question counts four marks, don't write down only one number or one word.
- If you have time, read your answers again and check them to see if they are correct and complete. For example, if an answer gives a length, make sure you have added the unit (mm, cm, m or km).

Understanding the following words can help you interpret and answer questions correctly:

compare: describe what is similar and different between two or more things
describe: give details and facts in full sentences without giving reasons
determine: find out
display: show
formulate: write down an idea or hypothesis that explains the idea clearly
identify: find, name and mention
illustrate: give an example of what you mean or explain it visually
investigate: follow a systematic way of analysing a problem
select: choose
sort: order information alphabetically, numerically, by date or importance

Glossary

acceleration: the rate at which an object's velocity changes, *98*
algorithm: a set of operations that produce a result, *158*
as the crow flies: shortest distance between two places, *70*
aviation: the activity or business of operating and flying aircraft, *118*
base: the number that is raised to an index, *16*
brokerage fee: the fee charged by a stockbroker for buying and selling shares and bonds, *114*
coincide: identical, synchronise, correspond, match, *137*
commutative: when changing the order of the numbers in an operation does not change the answer (for example, $2 \times 3 = 3 \times 2$), *51*
company: a structured commercial business or corporation, *113*
deceleration: negative acceleration (or slowing down), *98*
denominator: number under the line in a fraction, *38*
dividends: the part of a company's profit that is paid to shareholders on a regular basis, *113*
divisor: term that follows the division sign, *41*
hardware: physical components of a computer, *154*
index (or exponent): the number of times a real number is multiplied by itself, *16*
indices: plural of index, *16*
input: information that is needed to solve a problem, *158*
invert: turn upside down, *41*
like terms: the same variables (letters), *29*
matrices: plural of matrix, *47*
matrix: numbers arranged in rows and columns
number scale: a scale expressed in the form $a : b$ where a and b are numbers, *72*
numerator: number above the line in a fraction, *38*
output: the result of a calculation, *158*
p.a.: per annum (per year), *117*
place setting: the plates and cutlery for one person for a formal meal, *63*
power (of a number): the number raised to an index or exponent, *16*
prism: an object with two end faces that are similar and equal, and sides that are rectangles, *79*
quadratic: an expression in the form $ax^2 + bx + c$, *34*
reciprocal: the number by which a number is multiplied to give a product of 1 (example: $3 \times \frac{1}{3} = 1$), *19*
regular polygon: a closed 2D-shape with all its sides the same length and all angles the same size, *140*
regular polyhedron: a closed 3D-object with all its edges the same length; therefore all its faces are congruent, regular polygons; there are only five regular solids (the platonic solids), *140*
scalar: a quantity that has magnitude (size) only, *48, 89*
sequence: a structure in computing where statements are executed (carried out) one after the other, *159*
shareholders: the people who have bought shares in a company, *113*
shares: parts of the capital a company offers for sale in the form of shares, *113*
software: the programs that are used by a computer, *154*
stockbroker: person who buys and sells shares and bonds for others, *114*
stock exchange: the financial market of a country where shares are traded, *13*
unhindered: without being stopped, *24*
unlike terms: different variables, *29*
variable: symbol for a number or for a few different numbers, *29*
vector: a quantity that has both magnitude (size) and direction, *89*

Mathematics LEARNER'S BOOK **10**

FREDERICK FINCH
MARY CHANDA NYIRENDA
with RIKA POTGIETER

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